Minor Containment and Disjoint Paths in almost-linear time

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based on joint work with Michał Pilipczuk and Giannos Stamoulis
from the University of Warsaw (accepted to FOCS 2024)

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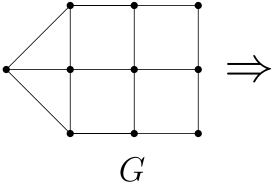

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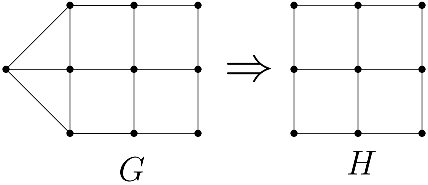

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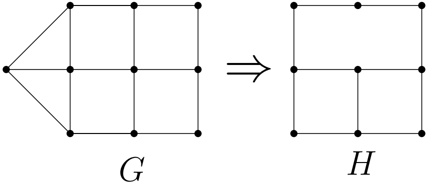

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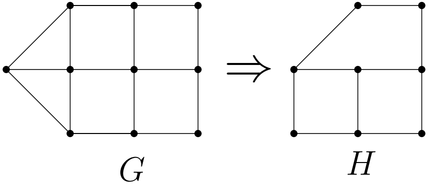

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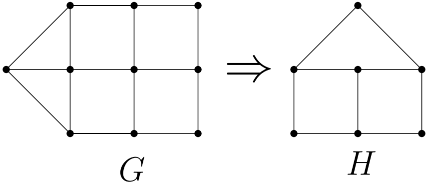

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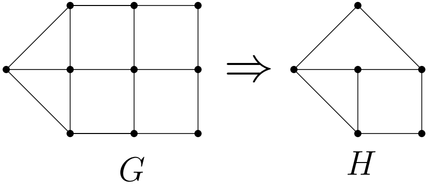

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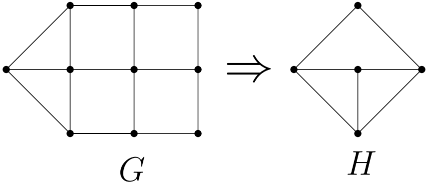

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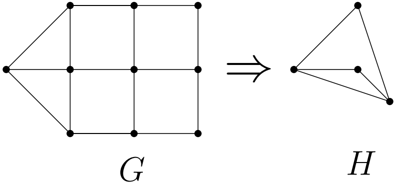

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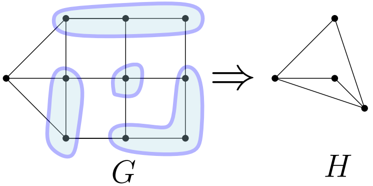

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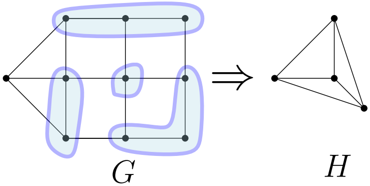

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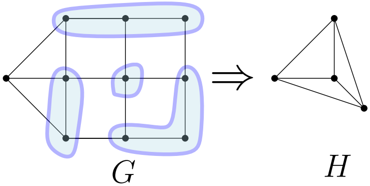

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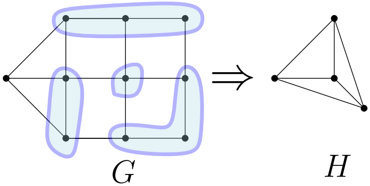

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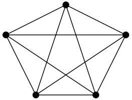
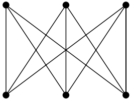
Planar $\Leftrightarrow$ no  or 

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The algorithm

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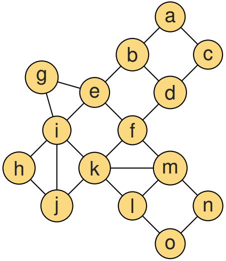

Graph G
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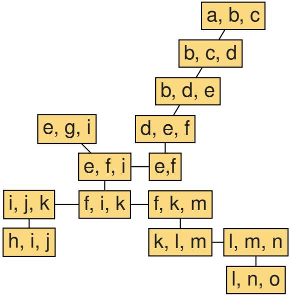

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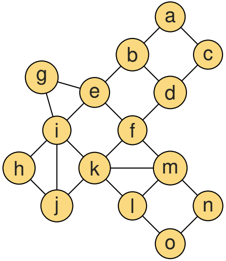

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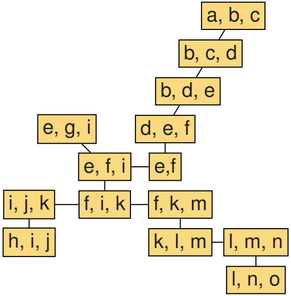

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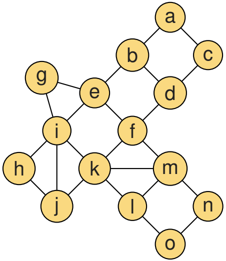

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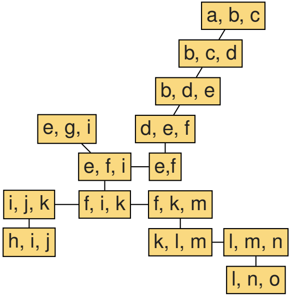

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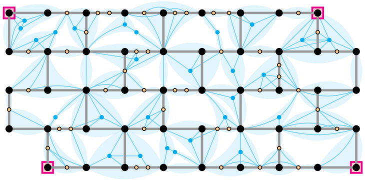

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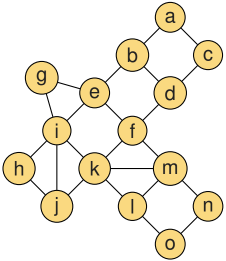

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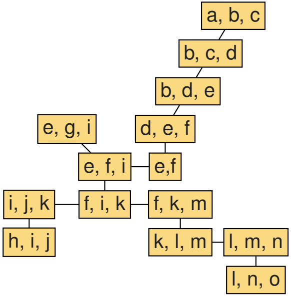

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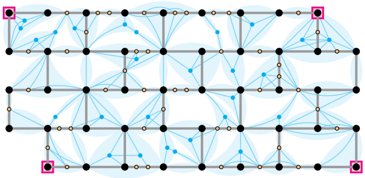

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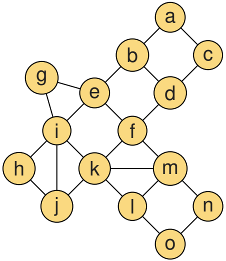

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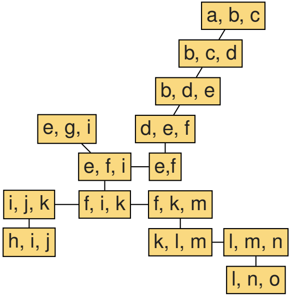

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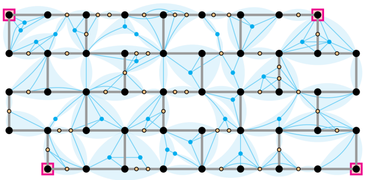

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- Computing the Robertson-Seymour decomposition, topological minor containment

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Thank you!