Certifying Automated Reasoning

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Helsinki Algorithms and Theory Days
August 30

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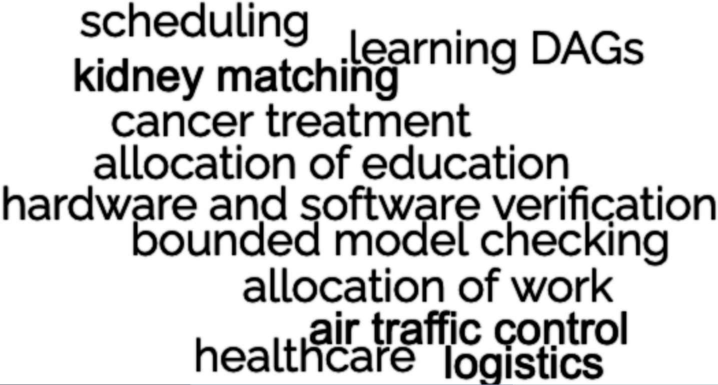
Automated reasoning

Significant progress in last couple of decades on combinatorial solvers

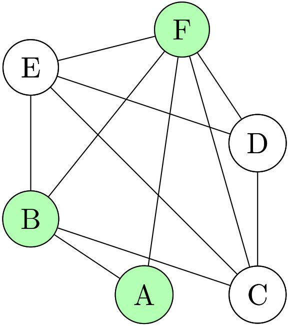
- Boolean satisfiability (SAT) & modulo theories (SMT), solving and optimization [Biere, Heule, van Maaren, and Walsh, 2021]
- Constraint programming [Rossi, van Beek, and Walsh, 2006]
- Pseudo-boolean (0-1 integer linear programming) [Elffers and Nordström, 2020].



Example problem

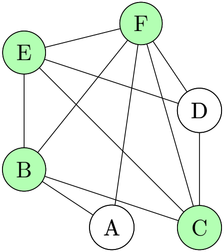
Maximum Clique

Decision problem: Is there clique of size 3? yes
Optimization problem: What is the size of the largest clique?



Automated Reasoning

for solving maximum clique



Maximize: $x_A + x_B + x_C + x_D + x_E + x_F$

subject to:

$$(1 - x_A) + (1 - x_C) \geq 1$$
$$(1 - x_D) + (1 - x_B) \geq 1$$
$$\vdots$$
$$x_j \in \{0, 1\} \quad \forall j$$

$\boxed{(1 - x) \equiv \neg x}$

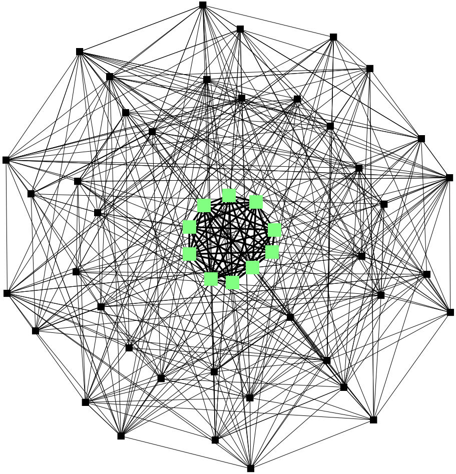
Problem instance → Constraint encoding

↓

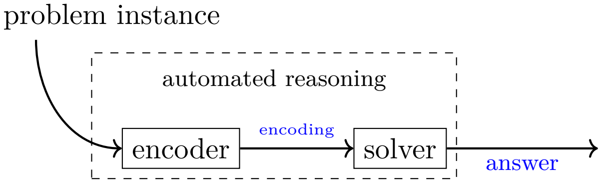
Solution to problem ← Solution to constraints
Green=1, Red=0

The main question of the day

Can we trust the answer?



Automated reasoning
in general

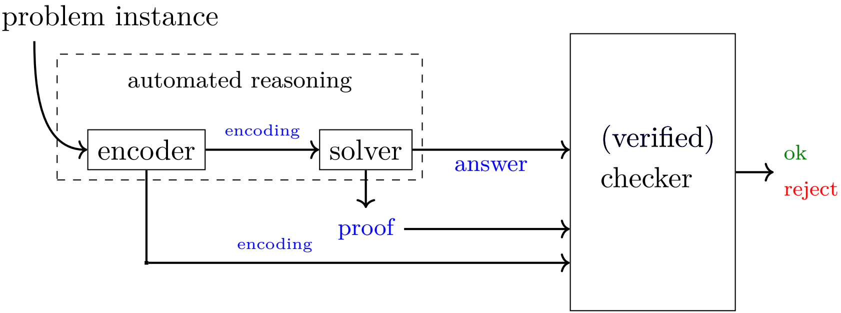


3 main approaches toward trustworthiness:

testing | formal verification | proof logging

Proof Logging

[Järvisalo, Heule, and Biere, 2012; Wetzler, Heule, and Jr., 2014; Heule, 2021; van Doornmalen, Eifler, Gleixner, and Hojny, 2023; Bogaerts, Gocht, McCreesh, and Nordström, 2022]



Desiderata of proof format

- powerful
- simple

Redundance-based proofs

Concrete Constraints

Propositional Logic, SAT, MaxSAT

- Instance:
  - Set of clauses, (CNF formula)
  - a linear objective function cost
- Find assignment τ that:
  - satisfies all clauses and
  - minimizes cost

$$\boxed{\begin{array}{l}\tau(y) = \tau(b_1) = \tau(b_3) = 1 \\ \tau(x) = \tau(b_2) = 0\end{array}}$$

$$F = \{(b_1 \vee x), (\neg x \vee b_2), (b_2 \vee y), (\neg y, b_3)\}$$

$$\text{cost} \equiv 2b_1 + 4b_2 + b_3$$

$$\text{cost}(\tau) = 3$$

Clause Redundancy

[Järvisalo, Heule, and Biere, 2012; Heule, Kiesl, and Biere, 2020; Ihalainen, Berg, and Järvisalo, 2022]

Definition

Clause C is redundant for formula F and objective cost if

$$\text{minimum-cost}(F) = \text{minimum-cost}(F \wedge C)$$

(wrt. cost)

equisatisfiability a special case

Example:

$(x \vee b_1) \wedge (\neg x \vee b_2)$

$\text{cost} = b_1 + 2b_2$

$(\neg b_2)$ is redundant

$(\neg b_1)$ is not redundant

Characterizing redundancy

[Heule, Kiesl, and Biere, 2020; Ihalainen, Berg, and Järvisalo, 2022]

(informal) Theorem

C redundant for F and cost iff there exists a set of literals L_C that fixes any solution τ of F that falsifies C without increasing its cost.

Example

$F = (x \lor b_1) \land (\neg x \lor b_2 \lor b_1) \land F' \longleftarrow$ F' does not contain $b_1, b_2, \neg b_1$ or $\neg b_2$

$\mathrm{cost} = b_1 + 2b_2$

$C = (\neg b_2)$ is redundant

$L_C = \{\neg b_2, b_1\}$

If τ satisfies F but falsifies C then assigning $b_1 = 1$, $b_2 = 0$ and the rest according to τ satisfies $F \land C$ with cost less than τ.

Note: adding redundant clauses might change the set of solutions

A (very simplified) redundancy-based proof

e.g. [Heule, Kiesl, and Biere, 2020; Bogaerts, Gocht, McCreesh, and Nordström, 2022]

A proof for $F = \{C_1, \ldots, C_n\}$ and cost is a sequence:

$$C_1, C_2, \ldots, C_n, C_{n+1}, \ldots [] \text{ =empty clause}$$

s.t. each C_{n+t} is either:

- redundant wrt. $C_1 \wedge \ldots \wedge C_{n+t-1}$, or
- $\text{cost} < \text{cost}(\tau)$ for a solution τ of $C_1 \wedge \ldots \wedge C_{n+t-1}$.

The proof establishes:

- optimality if $C_{n+t} = \text{cost} < \text{cost}(\tau)$ for some t
- infeasibility of constraints, otherwise

redundancy-based proof systems are strong
need to be careful with deletion

Redunancy for simulating solver reasoning

Subsumed Literal Elimination

[Berg, Saikko, and Järvisalo, 2016; Korhonen, Berg, Saikko, and Järvisalo, 2017]

Assume:

i) b_2 appears at least in the same clauses as b_1.

ii) the coefficient of b_2 in cost is at most the coefficient of b_1.

Then fix $b_2 = 0$ and simplify.

Reasoning

$\text{cost} = b_1 + 2b_2$

$(x \vee b_1) \wedge (\neg x \vee b_2 \vee b_1)$

$\downarrow$

$(x \vee b_1) \wedge (\neg x \vee b_1)$

$\downarrow$

(b_1)

$\downarrow$

$\emptyset$

Redundancy

$(x \vee b_1) \wedge (\neg x \vee b_2 \vee b_1)$

$\downarrow$

add $(\neg b_2)$

$\downarrow$

add (b_1)

$\downarrow$

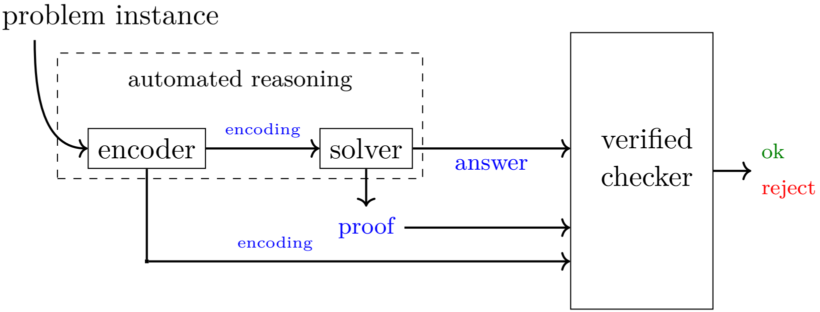
remove $(\neg x \vee b_1 \vee b_2)$

$\downarrow$

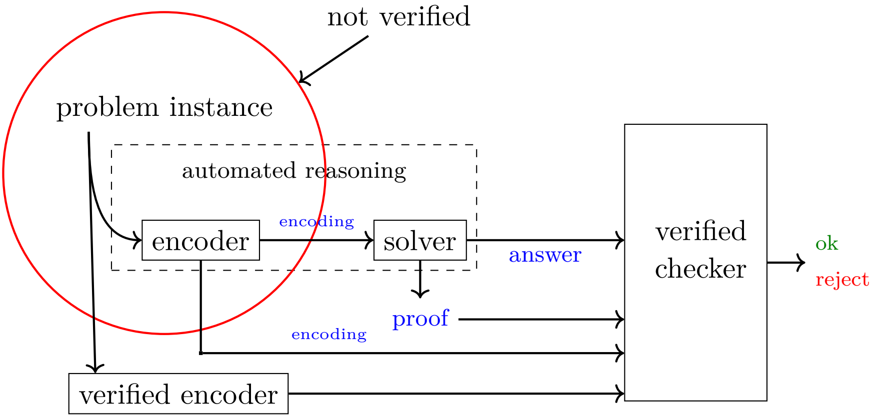
.

What do we need to trust?

Recap: Certified Automated Reasoning



What about the encoding?



- problem-specific verified encoder can prove the right properties of the encoding

What are these right properties?

[Gocht, McCreesh, Myreen, Nordström, Oertel, and Tan, 2024; Ihalainen, Oertel, Tan, Berg, Järvisalo, Myreen, and Nordström, 2024]

$$
\begin{aligned}
&\mathsf{is\_clique}\ vs\ (v,e) \stackrel{\text{def}}{=} \\
&\quad vs \subseteq \{\ 0,1,\ldots,v-1\ \} \wedge \\
&\quad \forall\, x\ y.\ x \in vs \wedge y \in vs \wedge x \neq y \Rightarrow \mathsf{is\_edge}\ e\ x\ y \\
&\mathsf{max\_clique\_size}\ g \stackrel{\text{def}}{=} \mathsf{max}_{\mathsf{set}}\ \{\ \mathsf{card}\ vs \mid \mathsf{is\_clique}\ vs\ g\ \}
\end{aligned}
$$

What are we trusting now?

- e.g. HOL model of verified checkers and correspondence to real system
- HOL4 theorem prover, including logic, implementation, and execution environment [Slind and Norrish, 2008]

Proof logging in the Constraint Reasoning and Optimization Group

Earlier

- Fundamentals of redundancy notions in boolean decision problems (SAT) [Järvisalo, Heule, and Biere, 2012]

Currently

- Fundamentals of redundancy notions in boolean optimization (MaxSAT) [Berg and Järvisalo, 2019; Ihalainen, Berg, and Järvisalo, 2022]
- Certifying solvers and preprocessors [Ihalainen, Oertel, Tan, Berg, Järvisalo, Myreen, and Nordström, 2024; Berg, Bogaerts, Nordström, Oertel, and Vandesande, 2023]
- Multiobjective optimization [Jabs, Berg, Ihalainen, and Järvisalo, 2023]

Conclusion

Proof logging in automated reasoning:

- Guarantees correctness of results
- Supports development of increasingly complex reasoning into solvers.
- Provides audibility to third parties without access to the solver.

Open Challenges

- Practical scaling.
- Proof logging e.g. PSPACE-complete problems.
- Proving bounds on the proof systems used.

I am hiring someone to work on these kinds of topics!

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