

Model-Agnostic Approximation of Constrained Forest Problems

Corinna Coupette, Alipasha Montaseri and Christoph Lenzen

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Steiner Forest: Generalization of Steiner Tree

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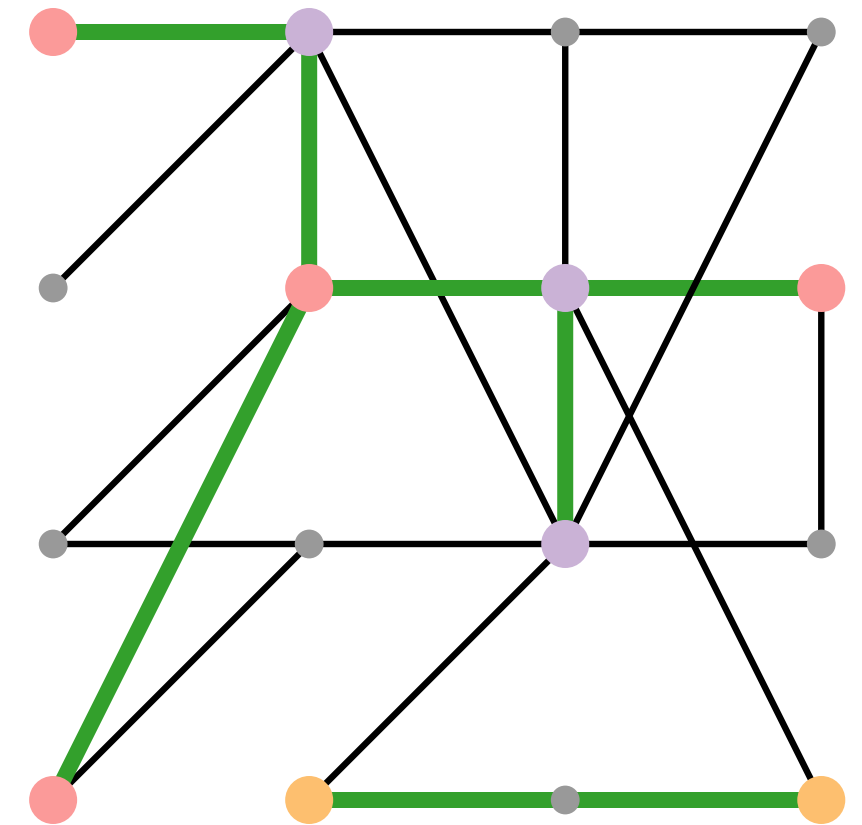
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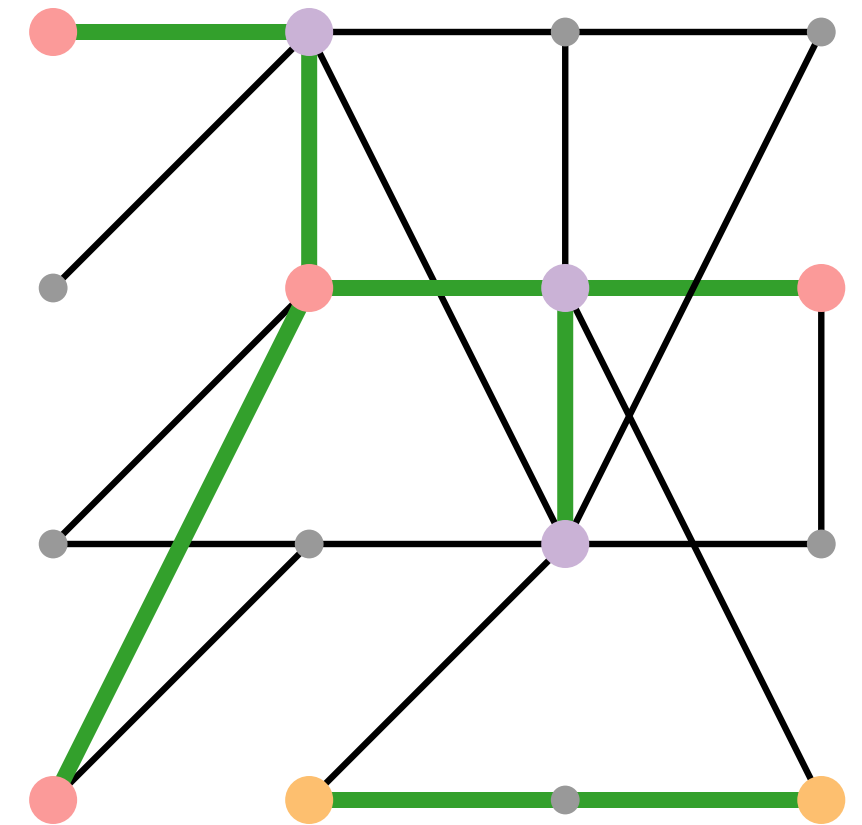


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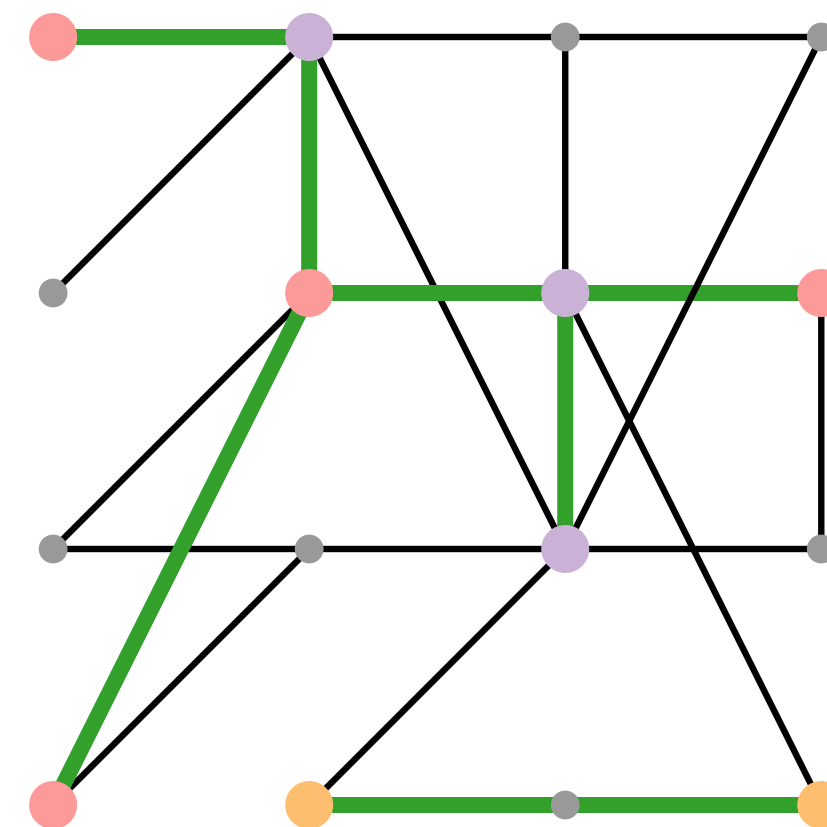
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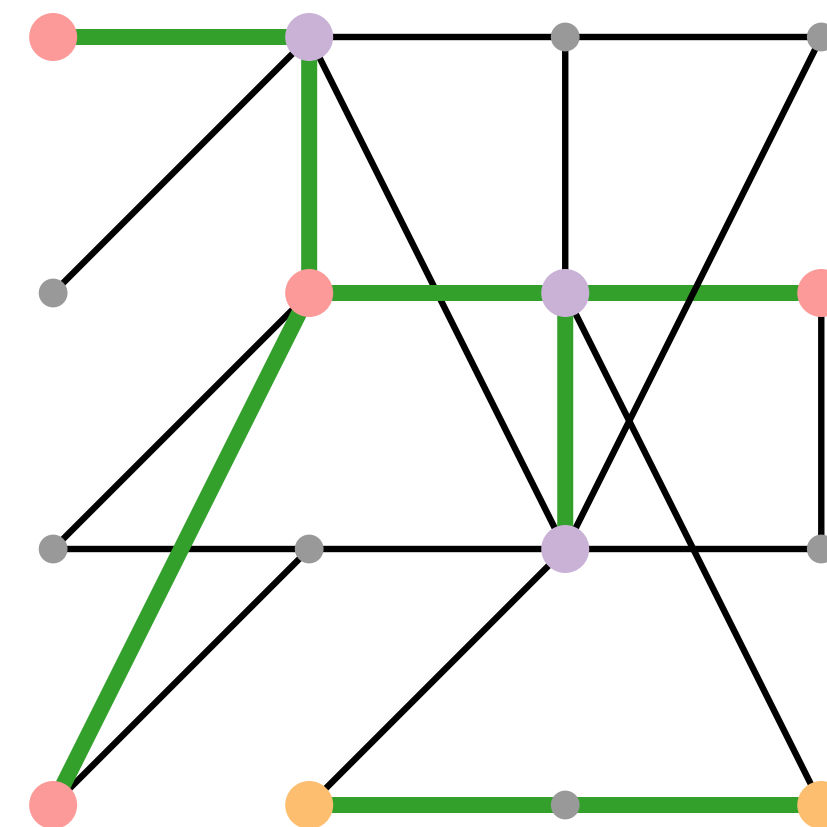
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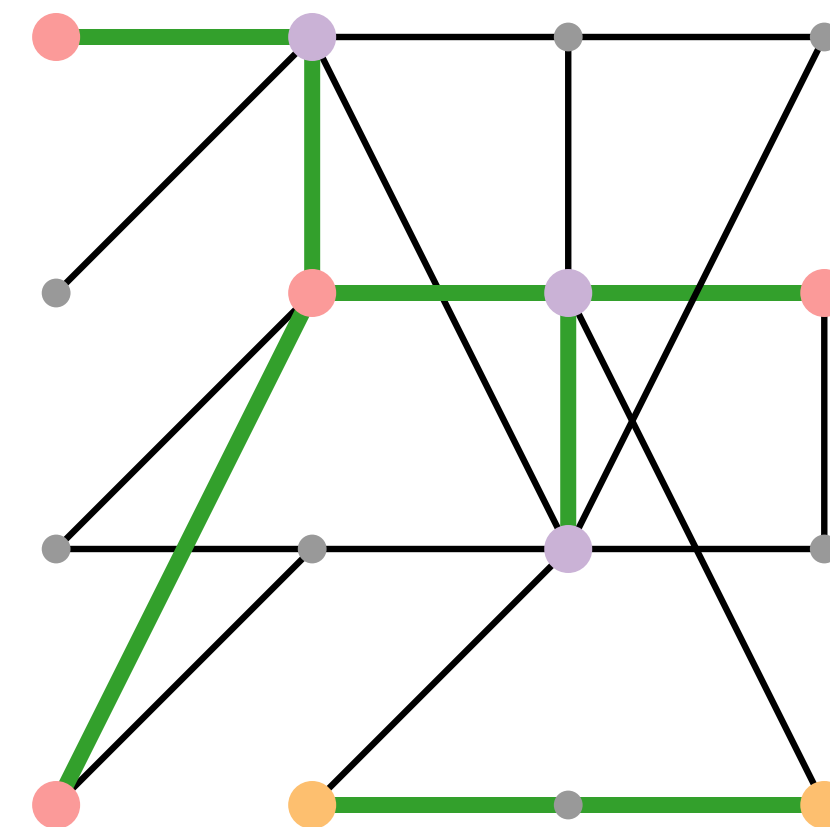
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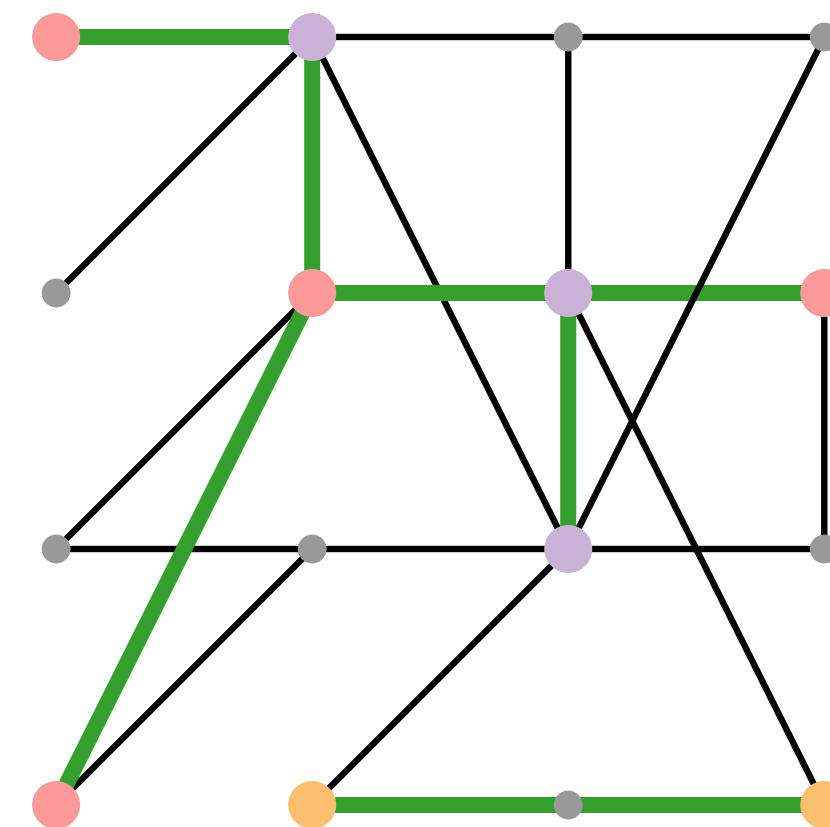
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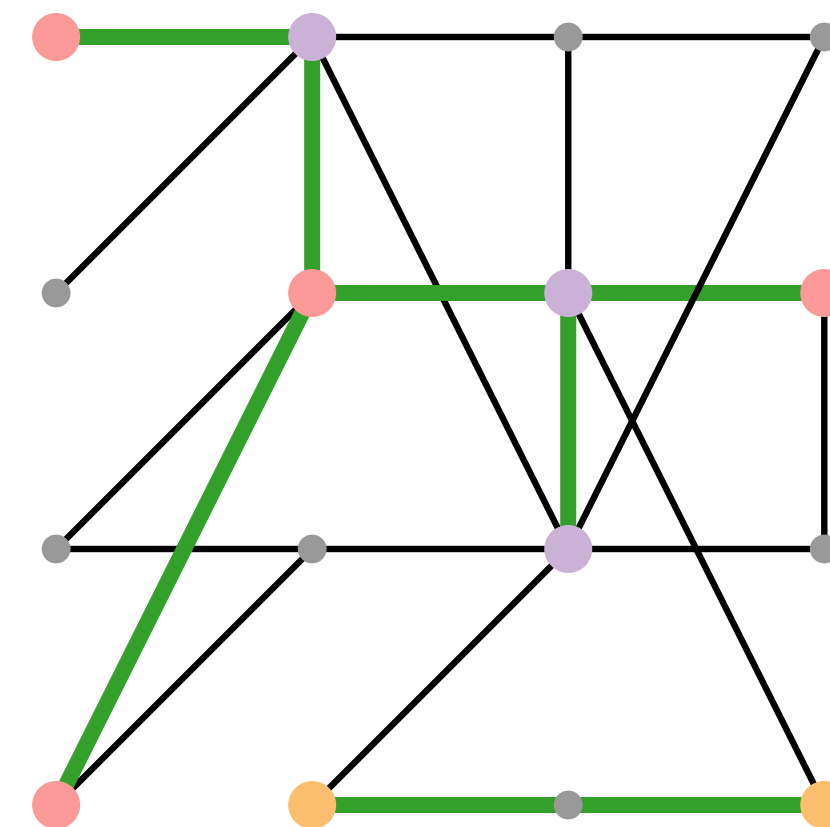
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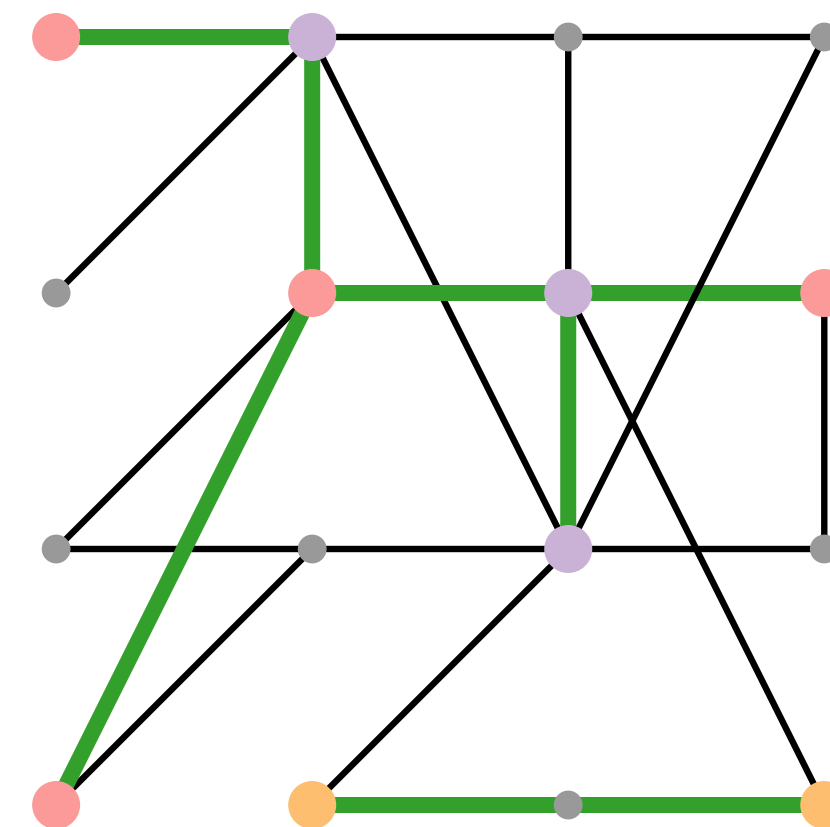
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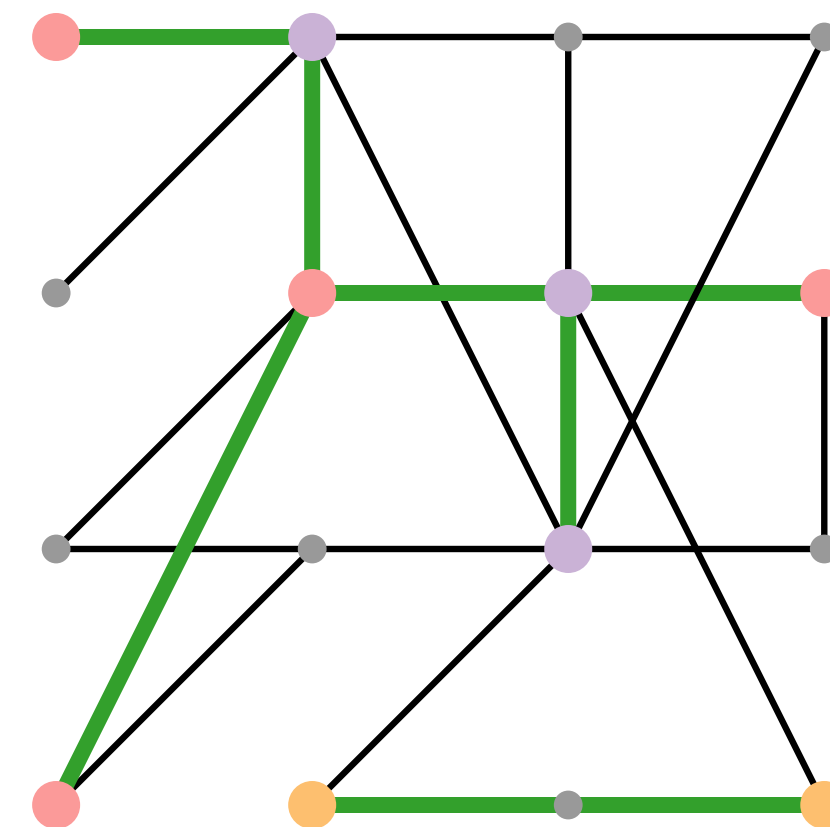
Challenges:

Model Specificity

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Existential Optimality

→ Universal Optimality (best on given topology)



Constrained Forest Problems (CFPs)

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≈ Problems on weighted graphs with forests as solutions

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Primal IP

$$\begin{aligned} & \min \sum_{e \in E} c(e)x_e \\ \text{s.t.} \quad & x(\delta(S)) \geq f(S) \quad \forall \emptyset \neq S \subset V \\ & x_e \in \{0, 1\} \quad \forall e \in E \end{aligned}$$

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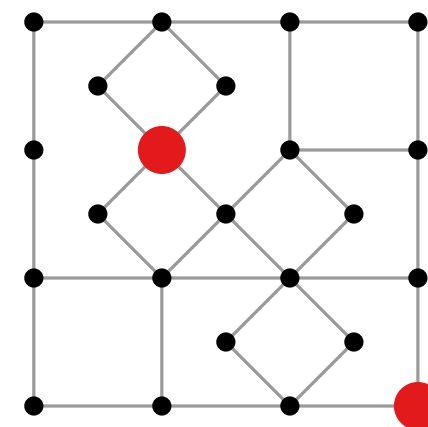
We focus on CFPs with *proper* functions f (zero, symmetry, disjointness)

Steiner Forest: $f(S) = 1 \Leftrightarrow \emptyset \neq S \cap V_i \neq V_i$ for some $i \in [k]$

The Goemans-Williamson Algorithm

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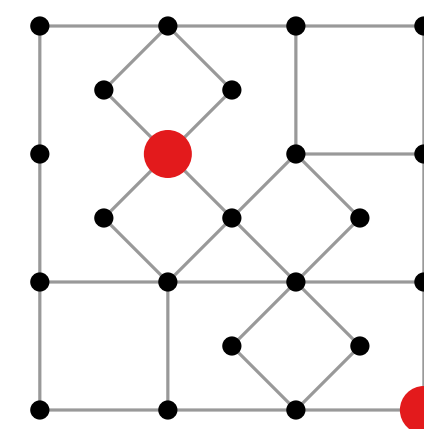
Input: Graph with edge costs c , proper forest function f



The Goemans-Williamson Algorithm

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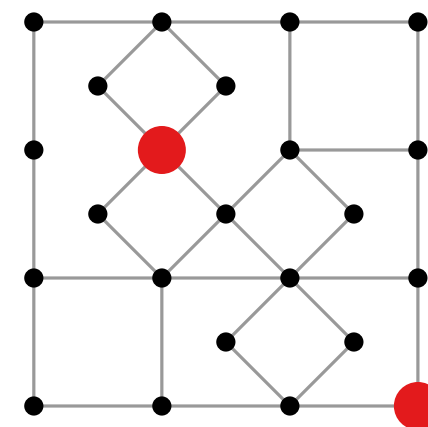


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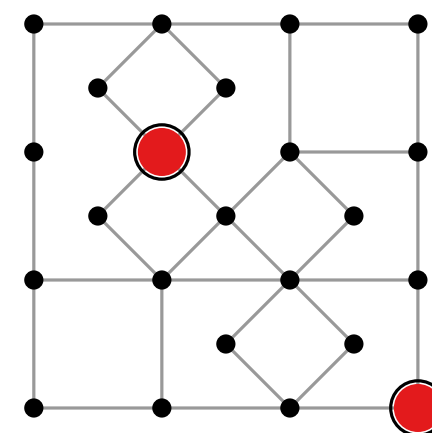
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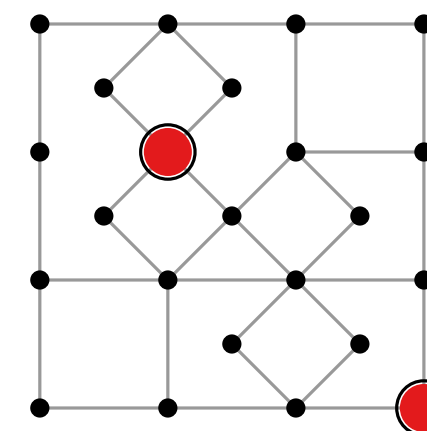
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While there are active components ($f(C) = 1$)



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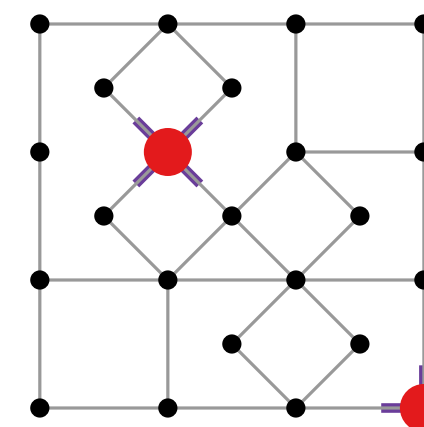
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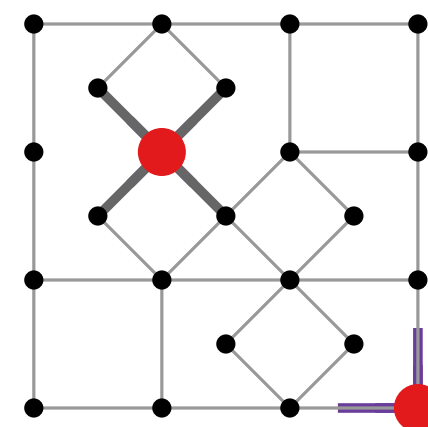
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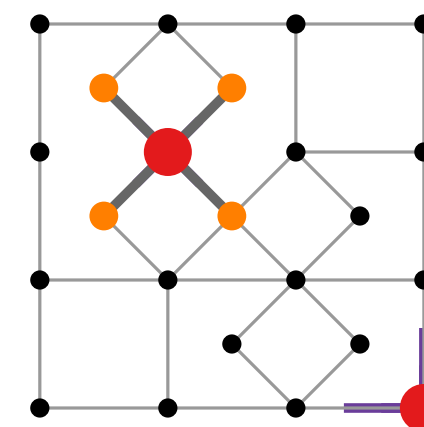
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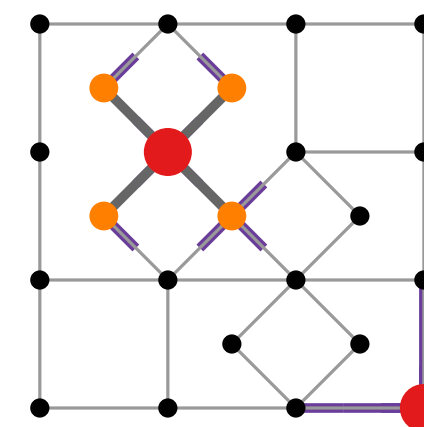
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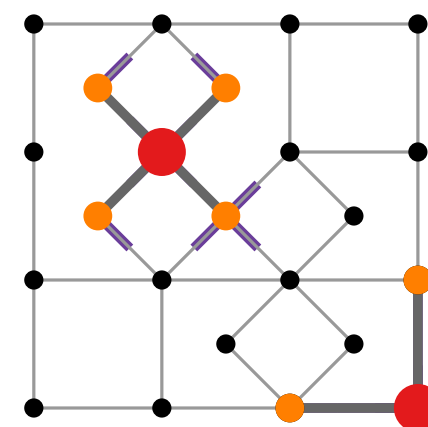
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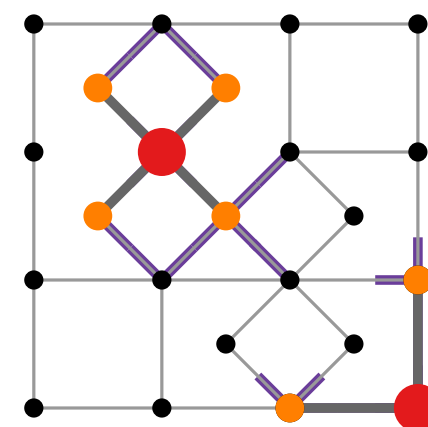
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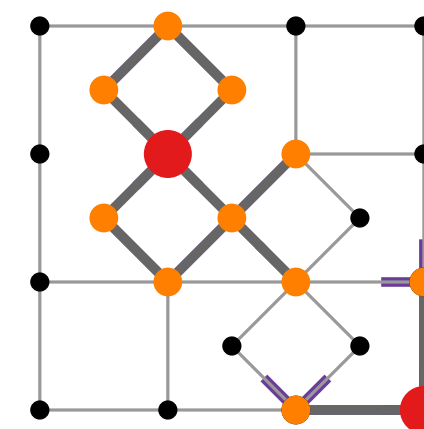
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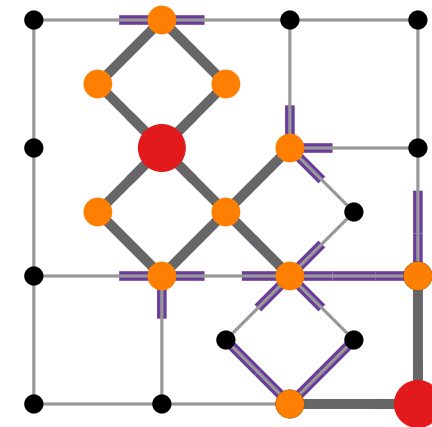
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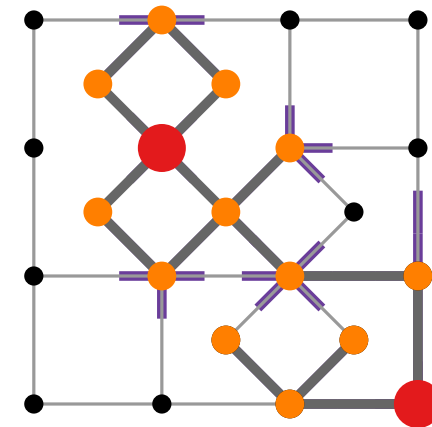
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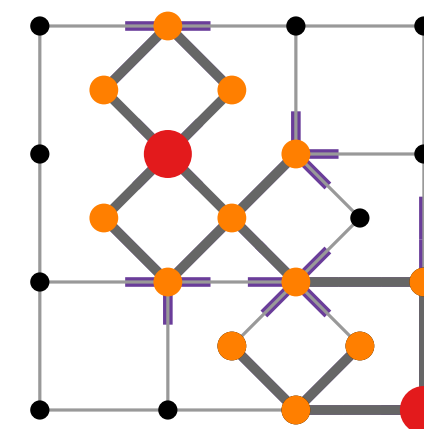
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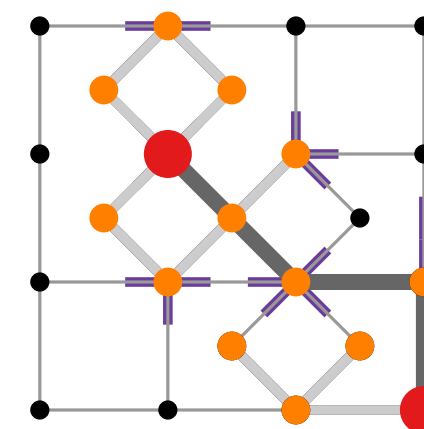
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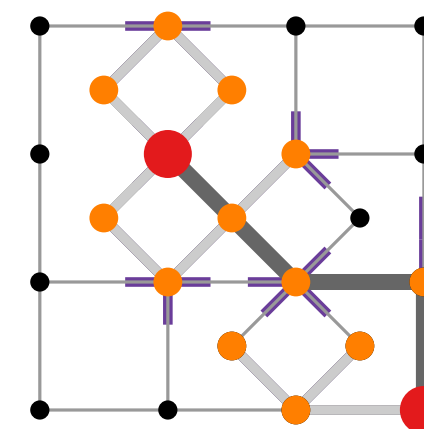
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Approximation Guarantee: $2 - 2/t$



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Challenge

Goemans-Williamson

Our Approach

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Our Approach

Solution-Set Construction

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Pairwise Distance Computations

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Forest-Function Evaluation

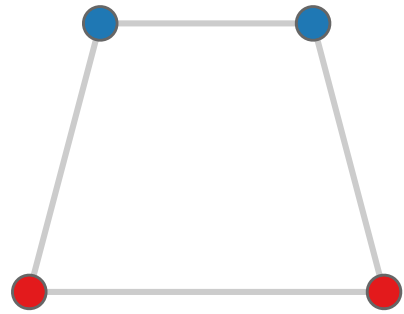
Many (Each Edge)

Deferred (Each Phase)

The Shell-Decomposition Algorithm: Approximation Intuition

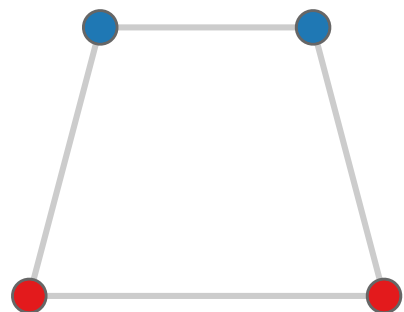
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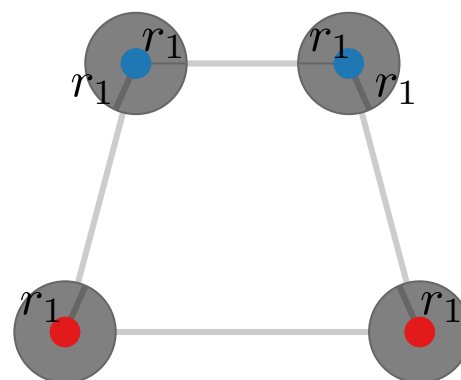


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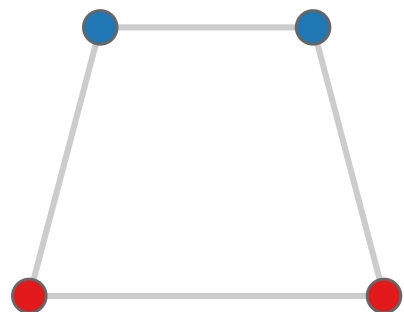


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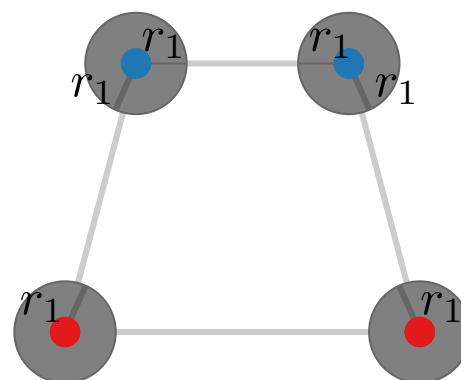


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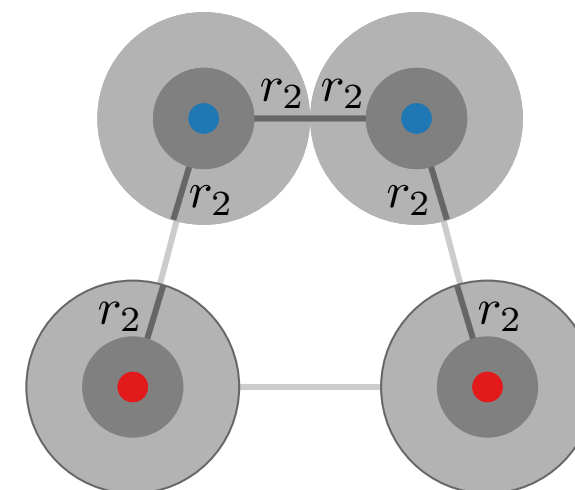
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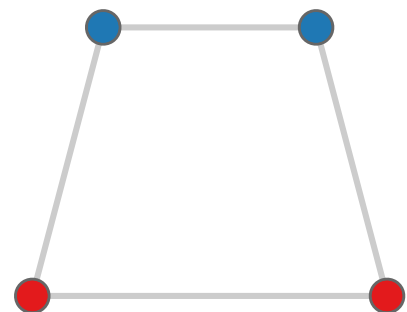


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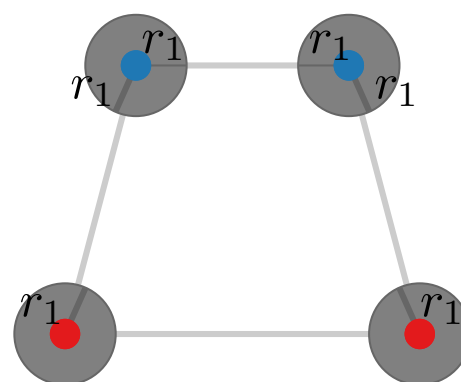


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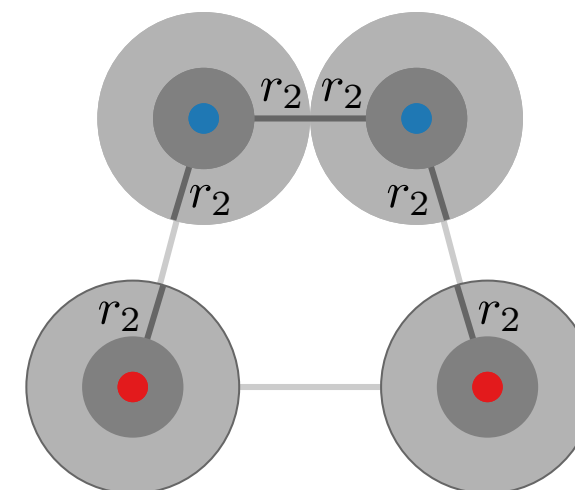
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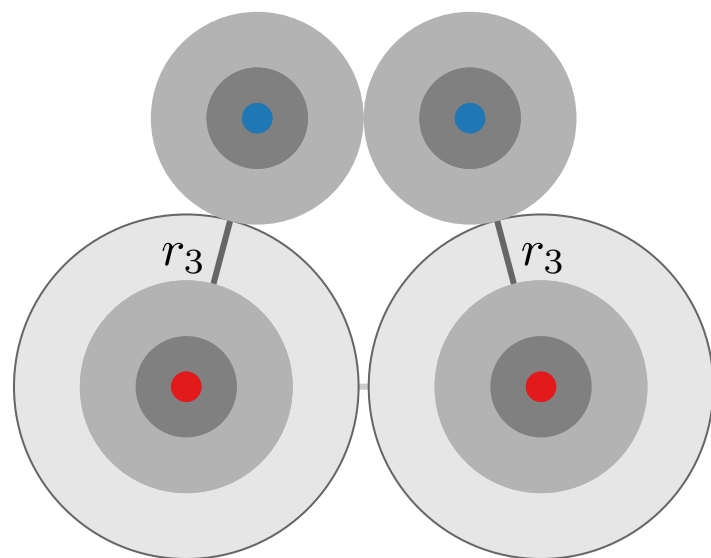
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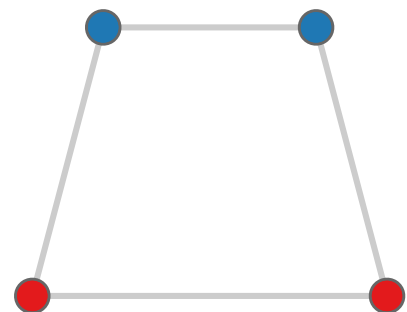


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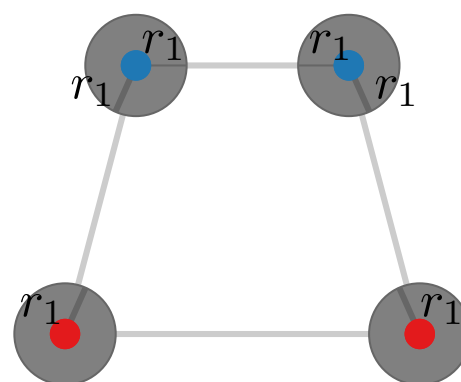


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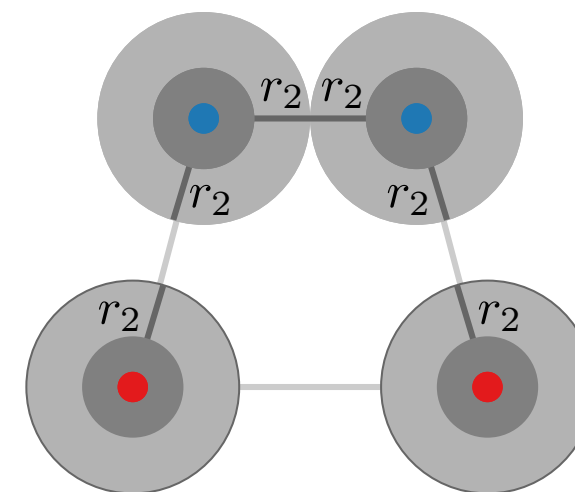
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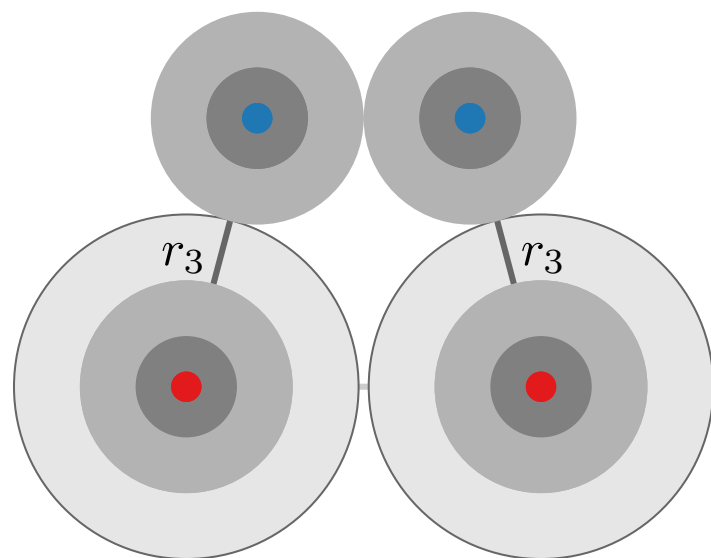
Phase 1



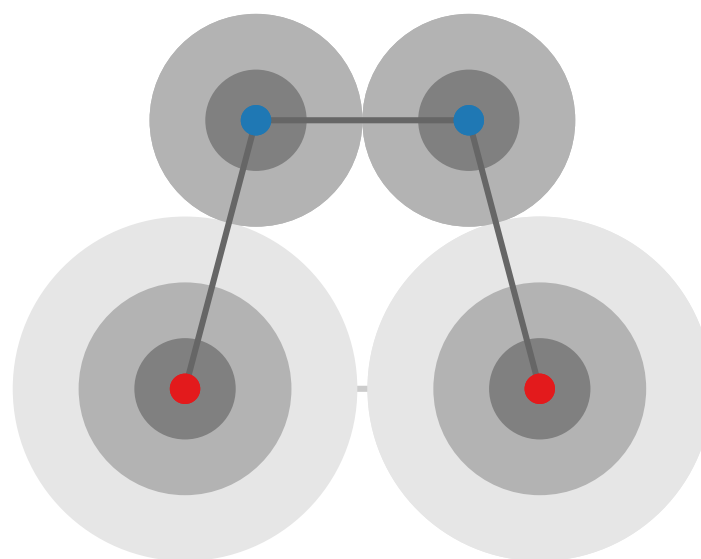
Phase 2



Phase 3

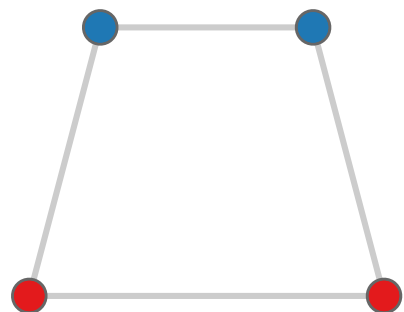


Identified Solution

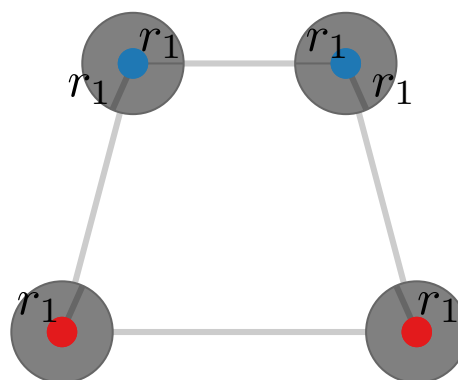


The Shell-Decomposition Algorithm: Approximation Intuition

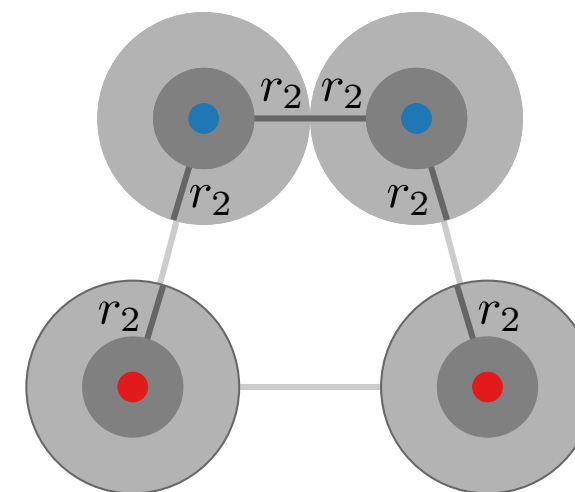
Start



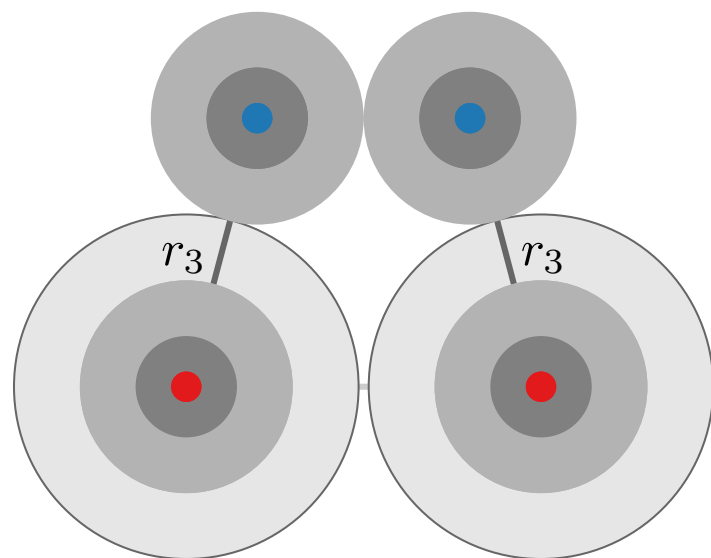
Phase 1



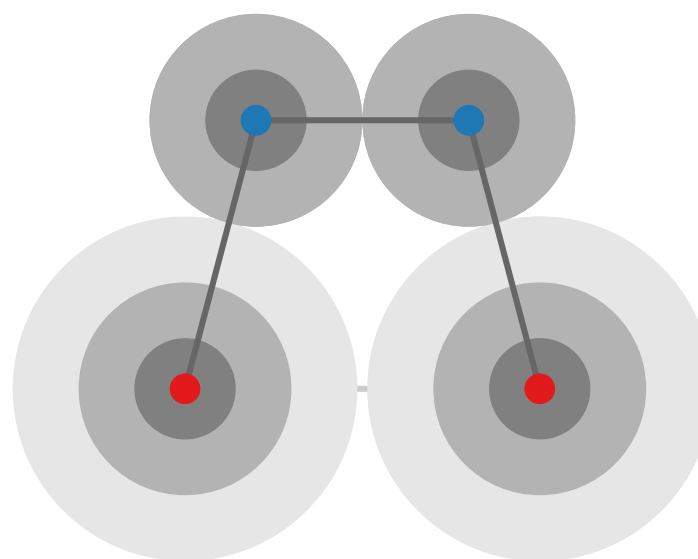
Phase 2



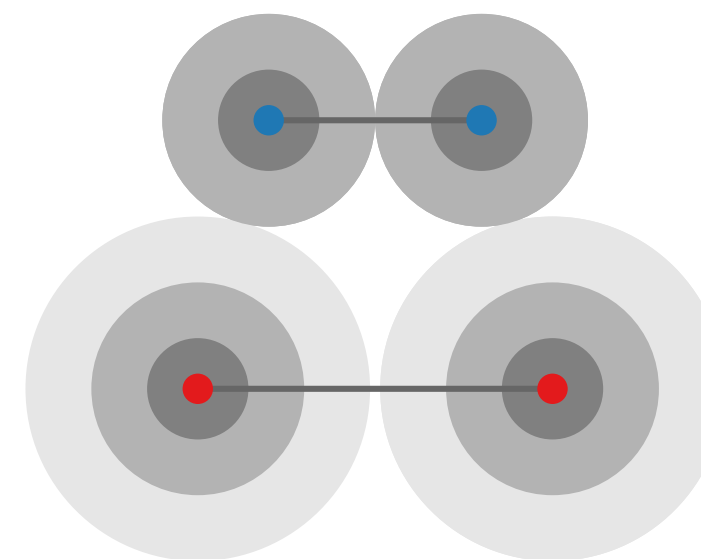
Phase 3



Identified Solution



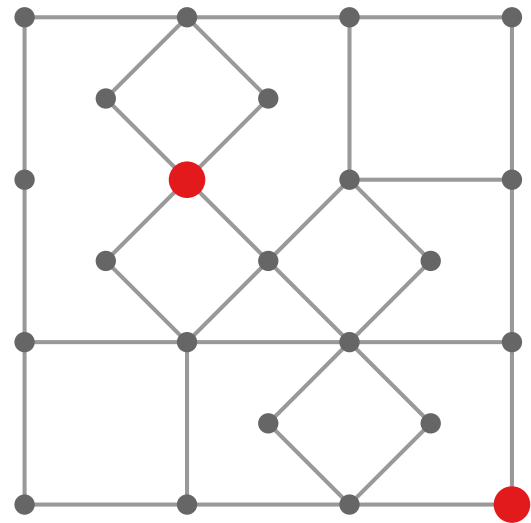
Optimal Solution



The Shell-Decomposition Algorithm: Building Blocks

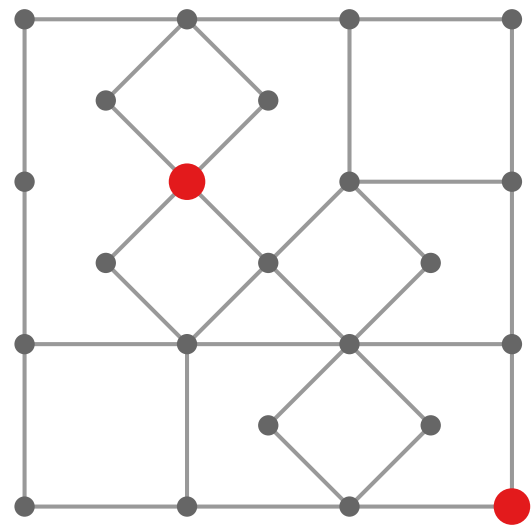
The Shell-Decomposition Algorithm: Building Blocks

Initialization

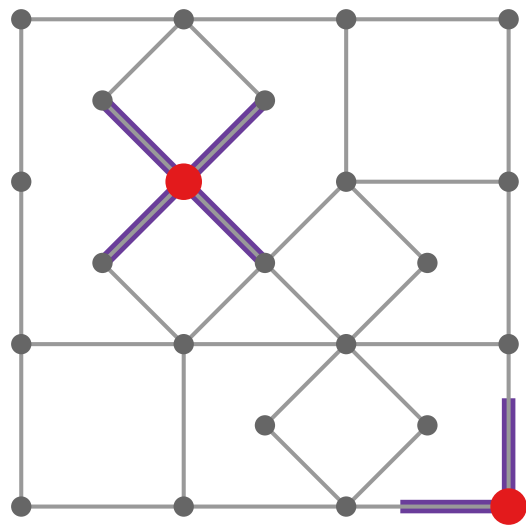


The Shell-Decomposition Algorithm: Building Blocks

Initialization

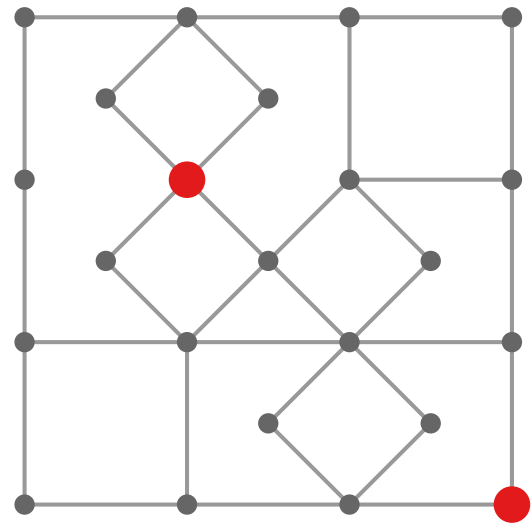


(0) Ball Growth

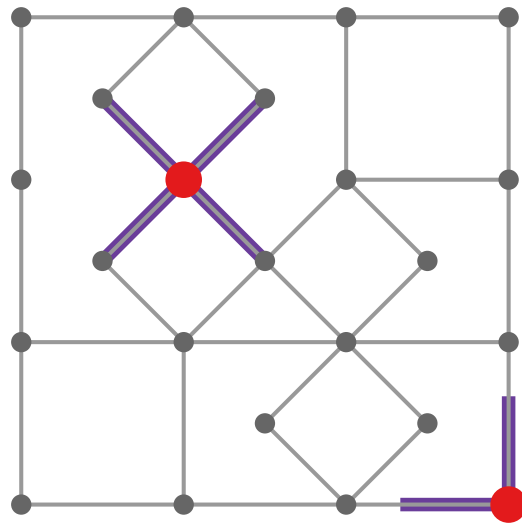


The Shell-Decomposition Algorithm: Building Blocks

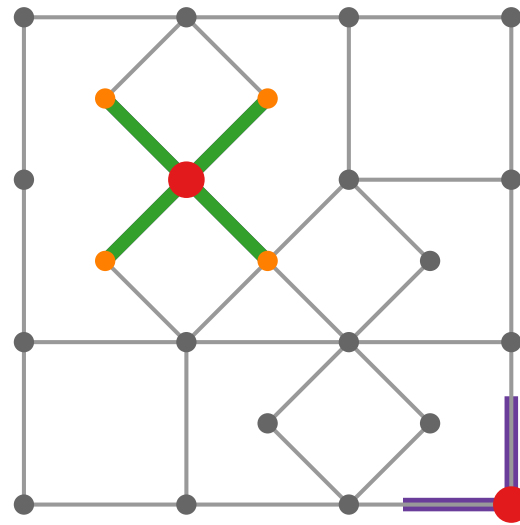
Initialization



(0) Ball Growth

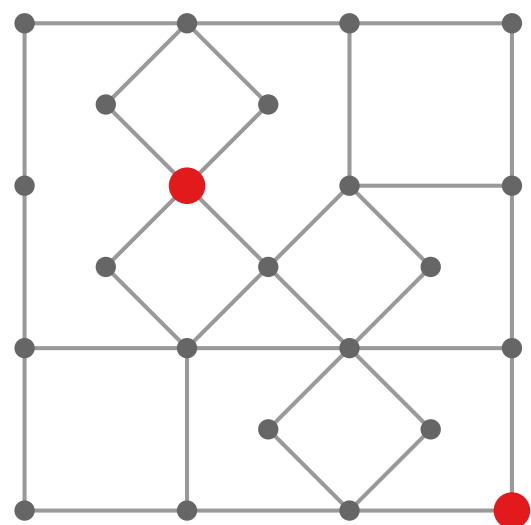


(0) SSSP Cover

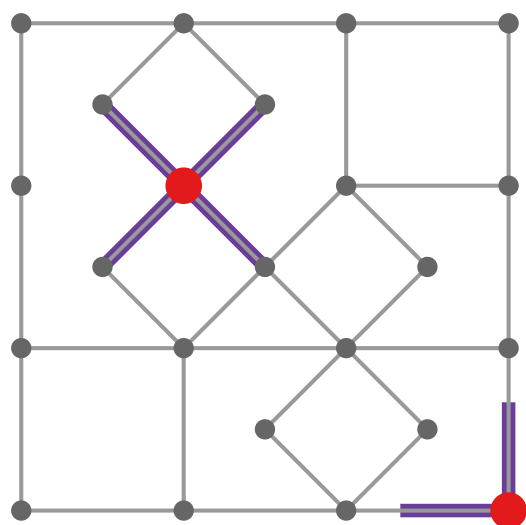


The Shell-Decomposition Algorithm: Building Blocks

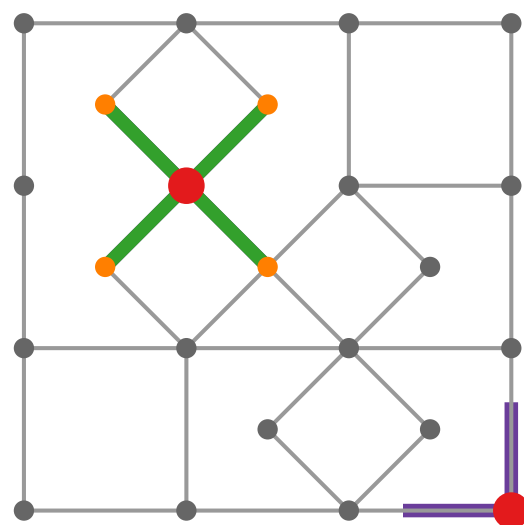
Initialization



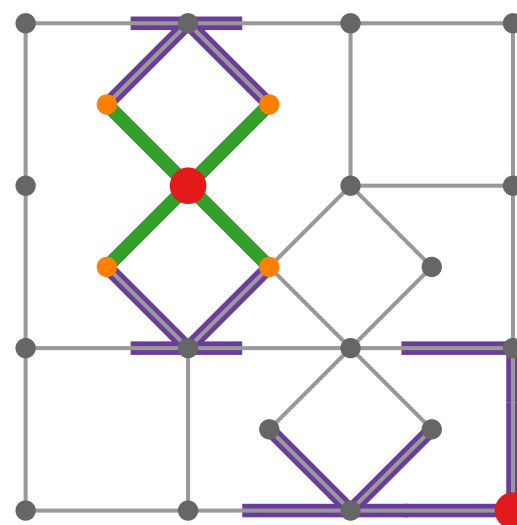
(0) Ball Growth



(0) SSSP Cover

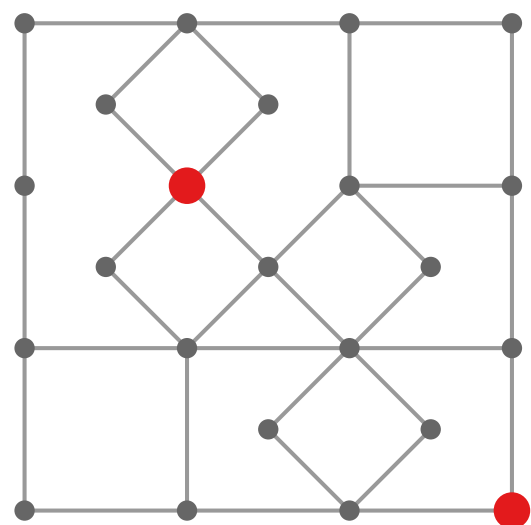


(1) Ball Growth

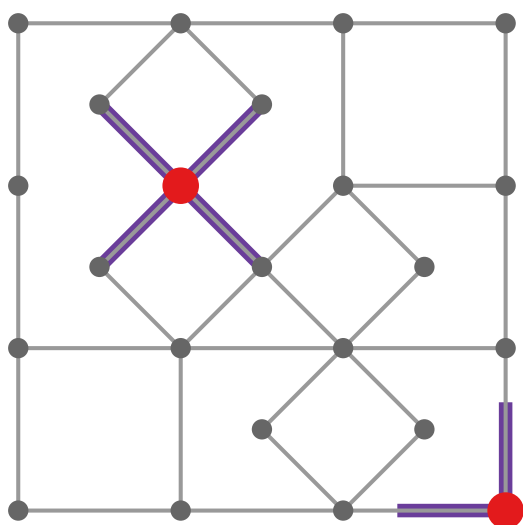


The Shell-Decomposition Algorithm: Building Blocks

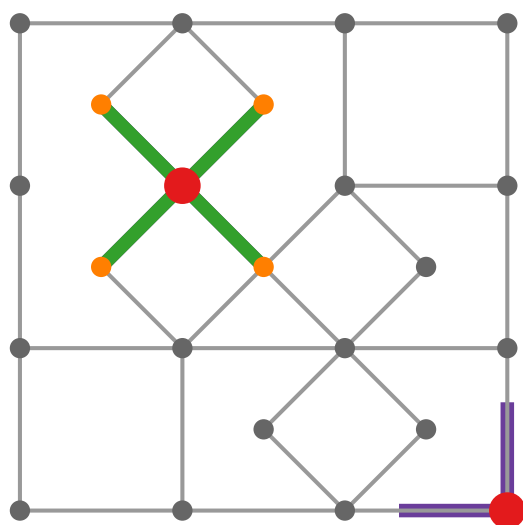
Initialization



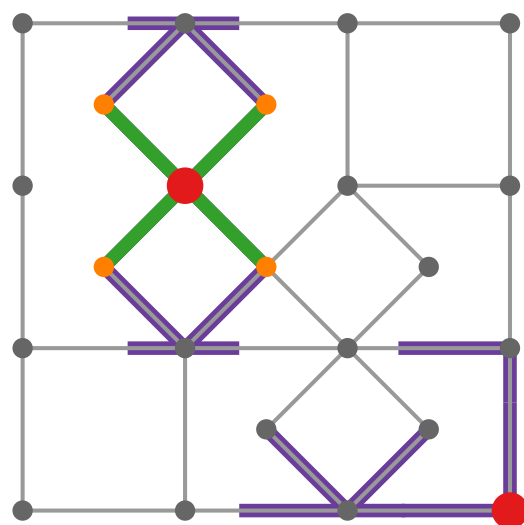
(0) Ball Growth



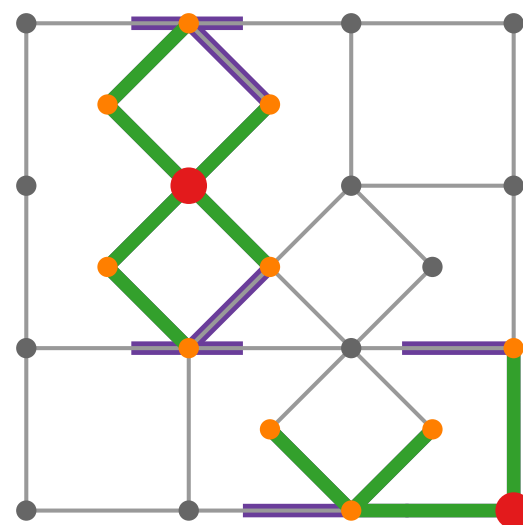
(0) SSSP Cover



(1) Ball Growth

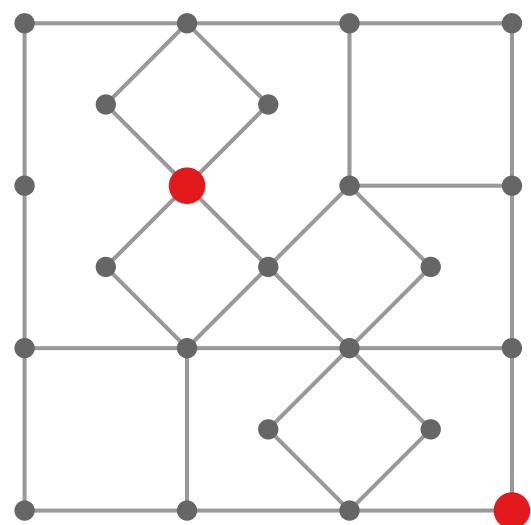


(1) SSSP Cover

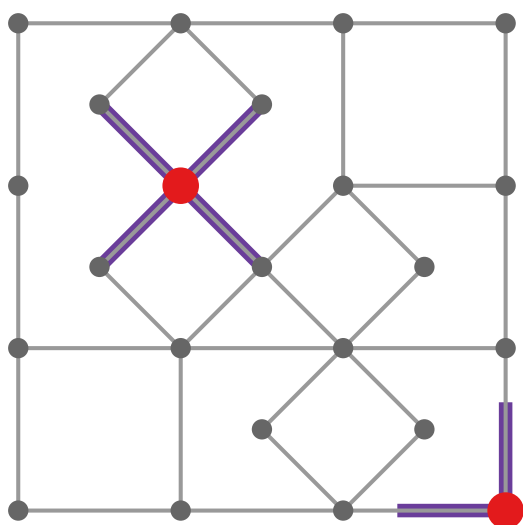


The Shell-Decomposition Algorithm: Building Blocks

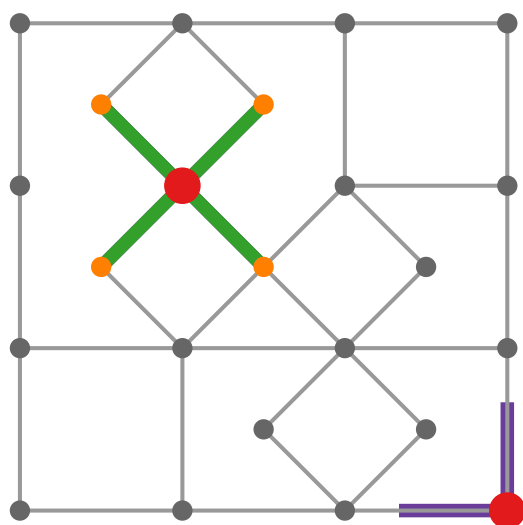
Initialization



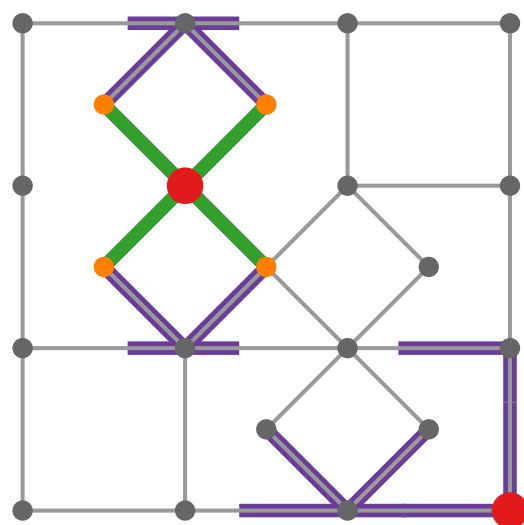
(0) Ball Growth



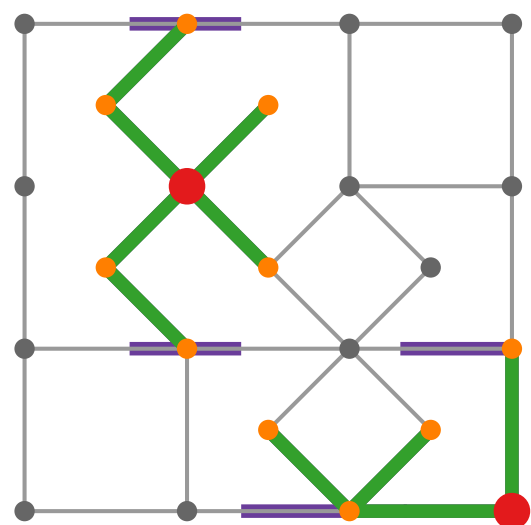
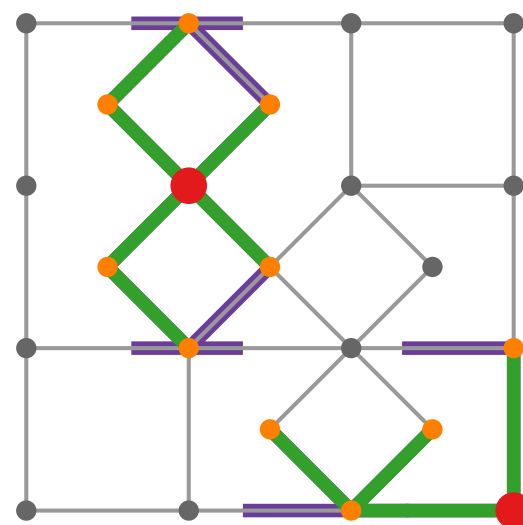
(0) SSSP Cover



(1) Ball Growth



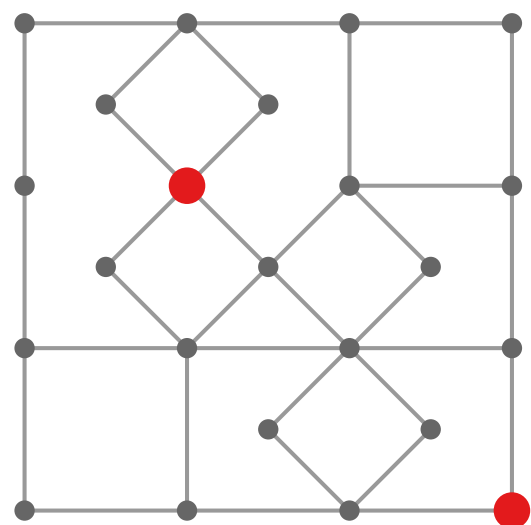
(1) SSSP Cover



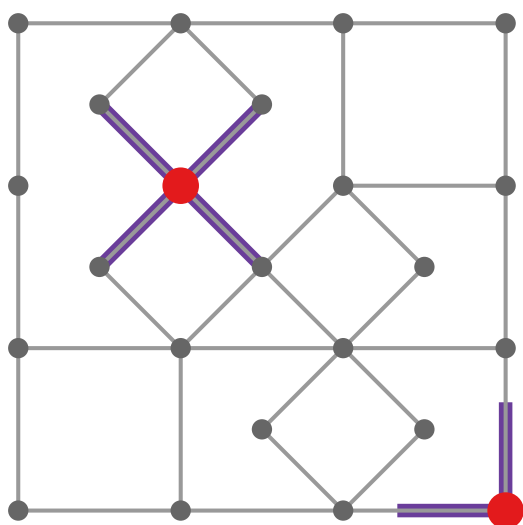
(1) Edge Deletion

The Shell-Decomposition Algorithm: Building Blocks

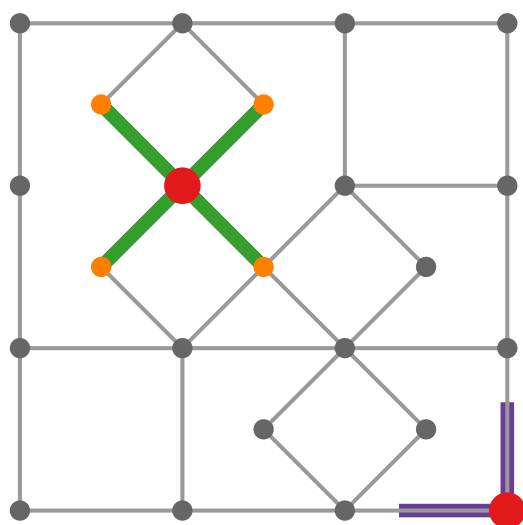
Initialization



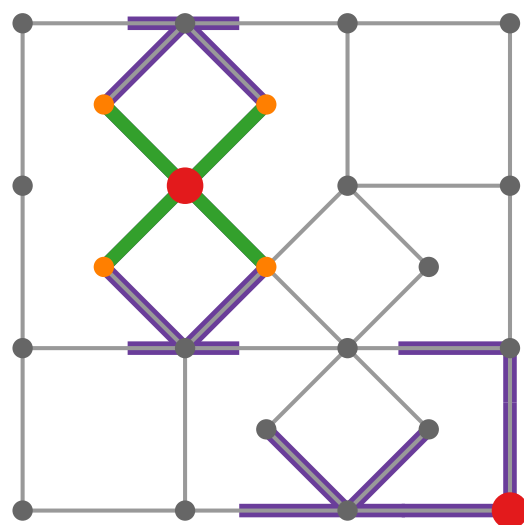
(0) Ball Growth



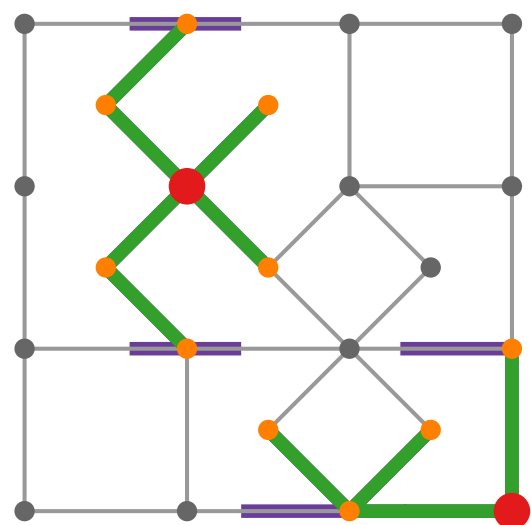
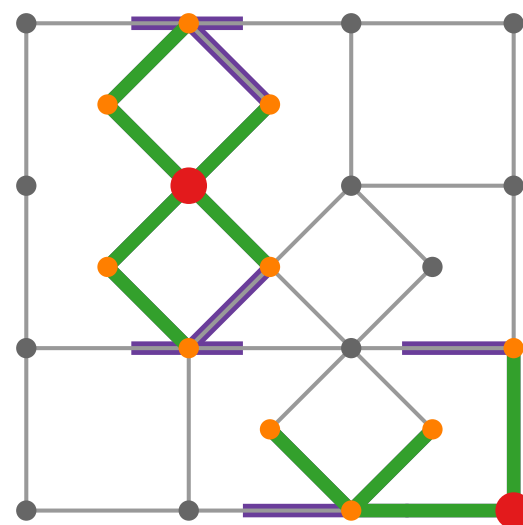
(0) SSSP Cover



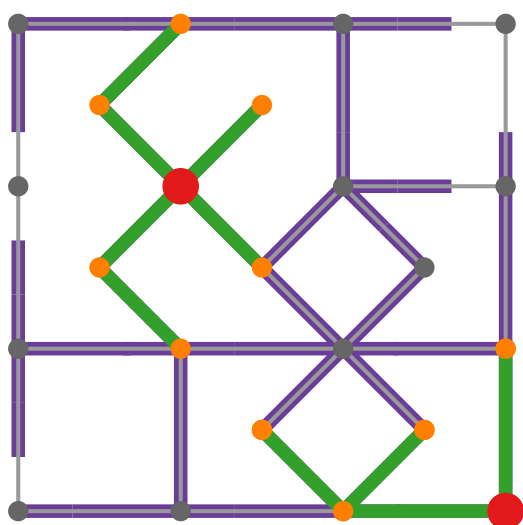
(1) Ball Growth



(1) SSSP Cover



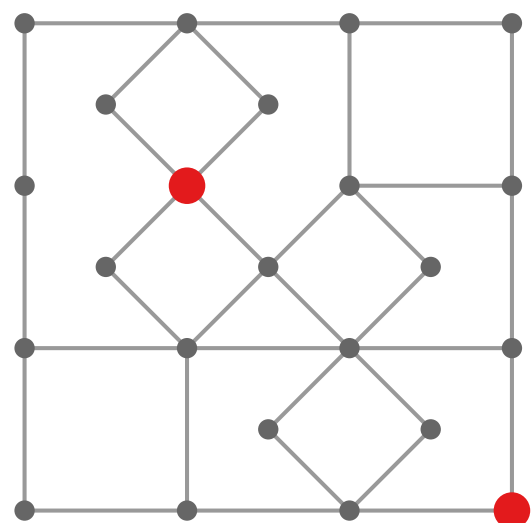
(1) Edge Deletion



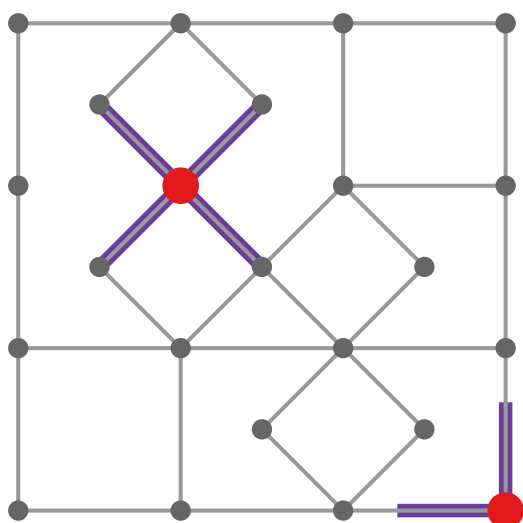
(2) Ball Growth

The Shell-Decomposition Algorithm: Building Blocks

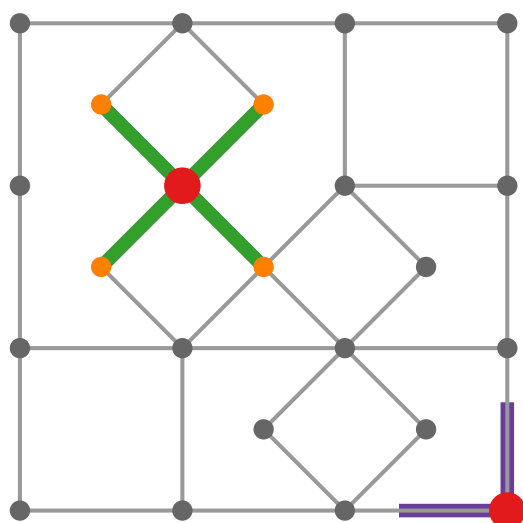
Initialization



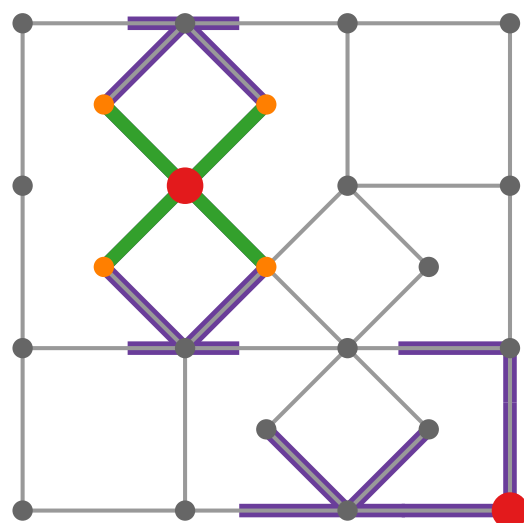
(0) Ball Growth



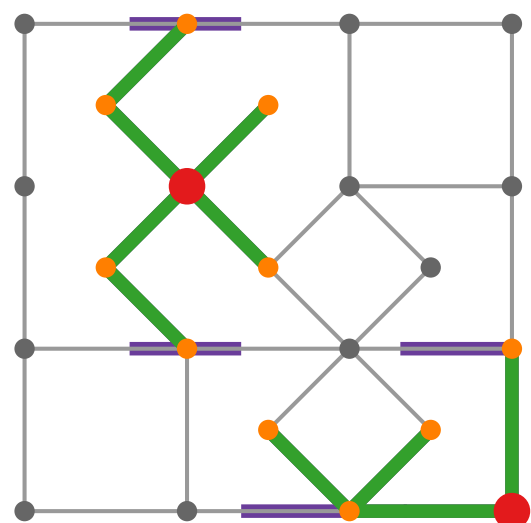
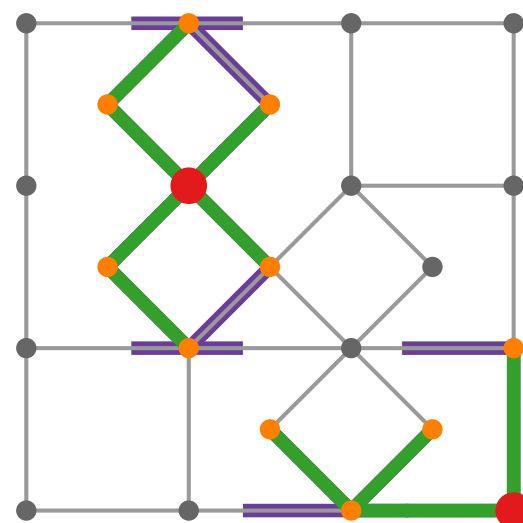
(0) SSSP Cover



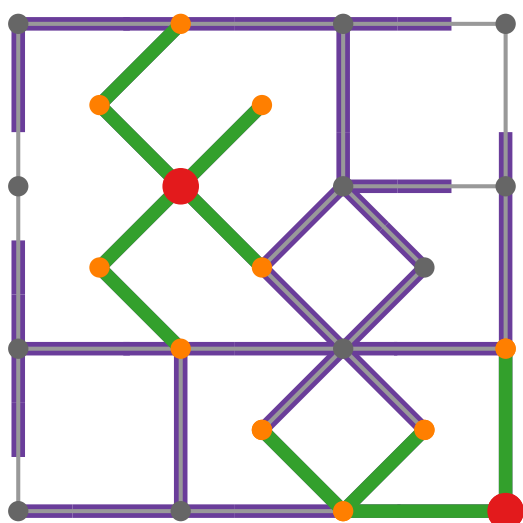
(1) Ball Growth



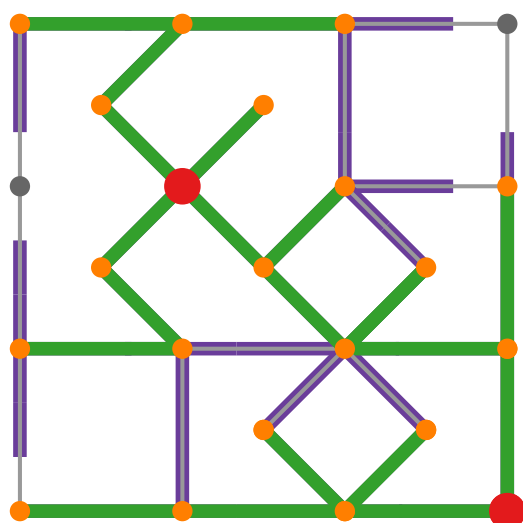
(1) SSSP Cover



(1) Edge Deletion



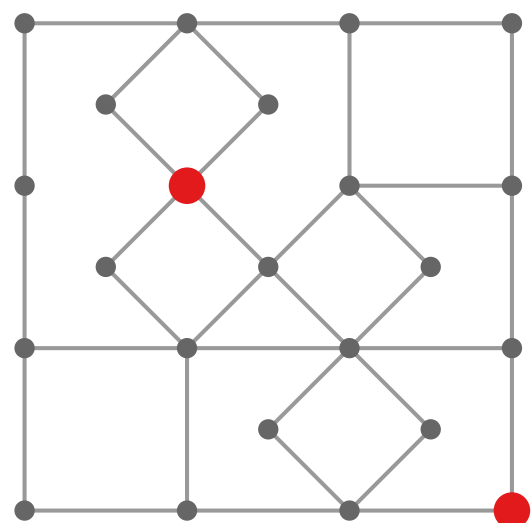
(2) Ball Growth



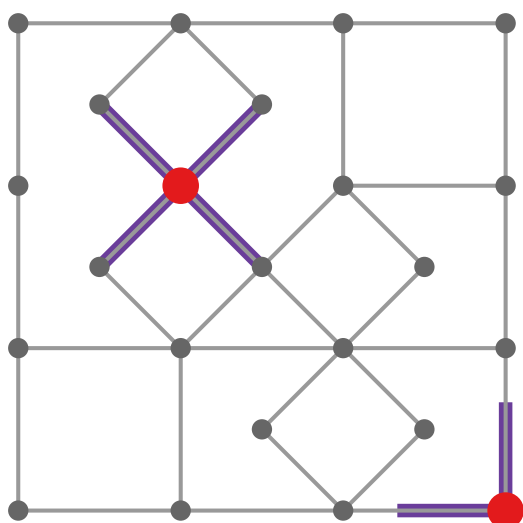
(2) SSSP Cover

The Shell-Decomposition Algorithm: Building Blocks

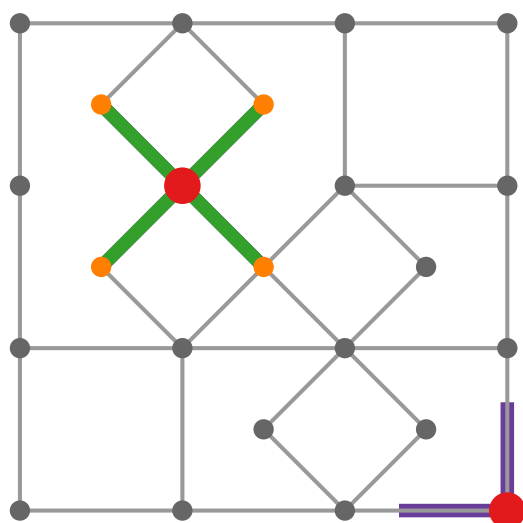
Initialization



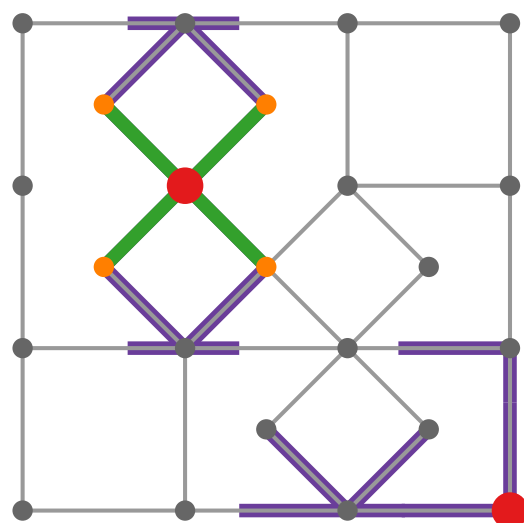
(0) Ball Growth



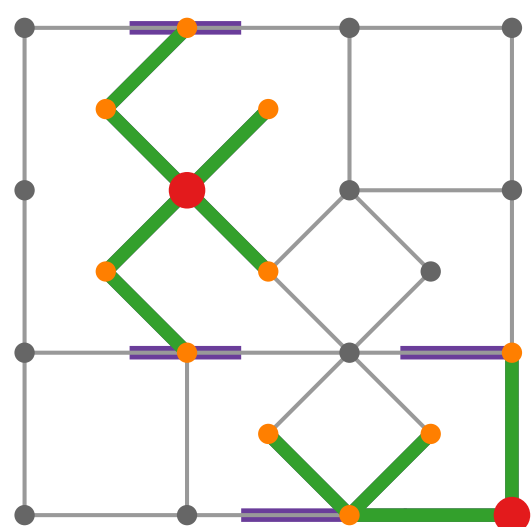
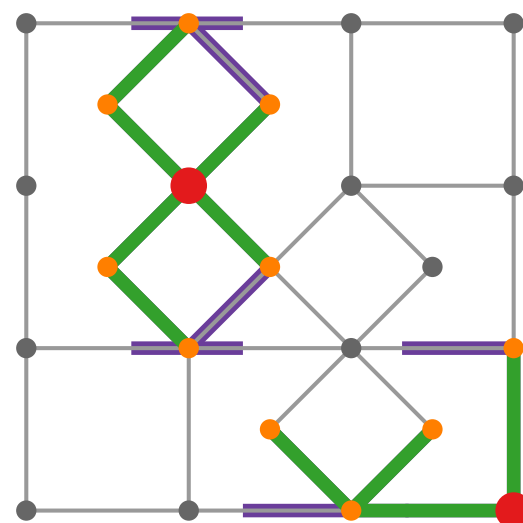
(0) SSSP Cover



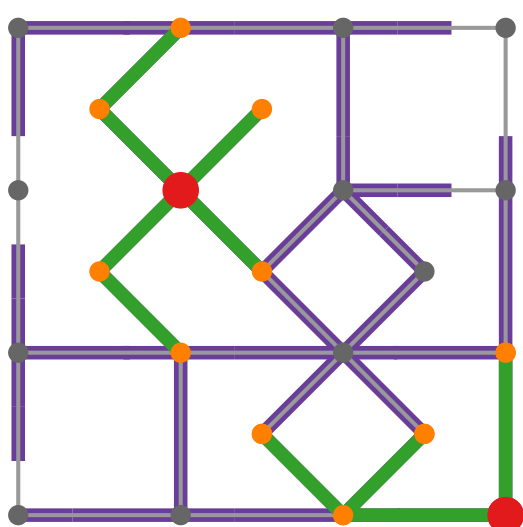
(1) Ball Growth



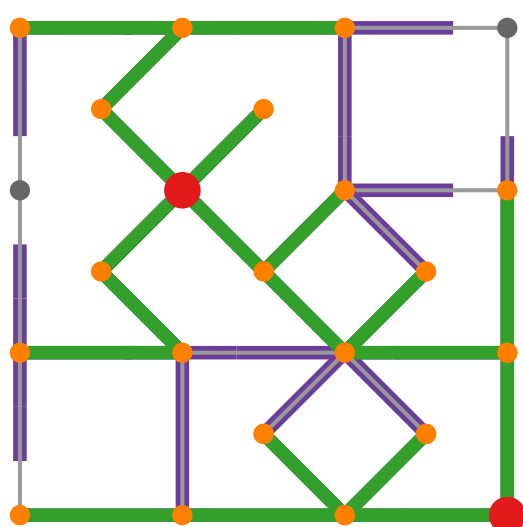
(1) SSSP Cover



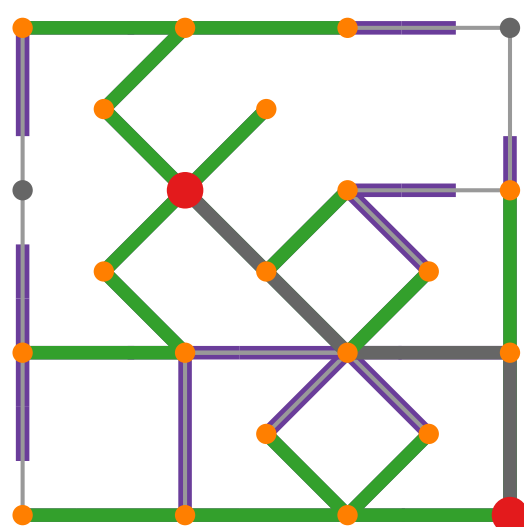
(1) Edge Deletion



(2) Ball Growth



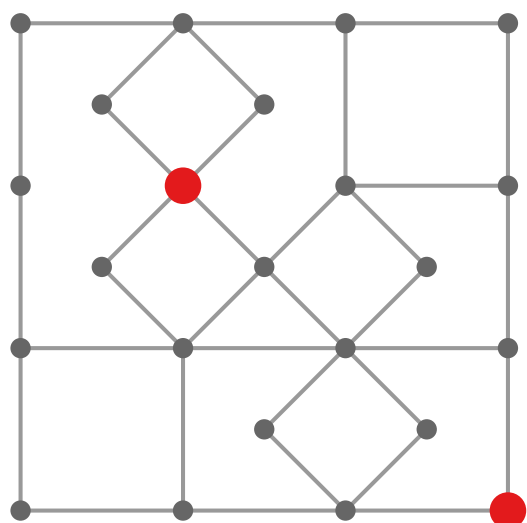
(2) SSSP Cover



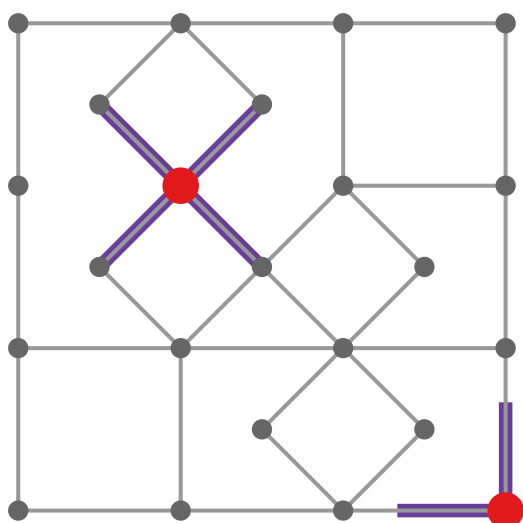
(2) Merge Execution

The Shell-Decomposition Algorithm: Building Blocks

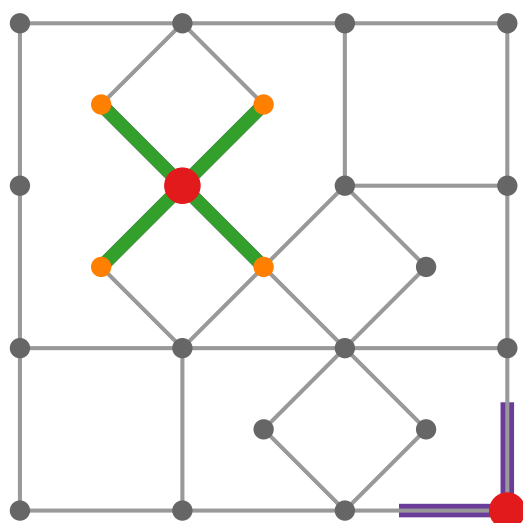
Initialization



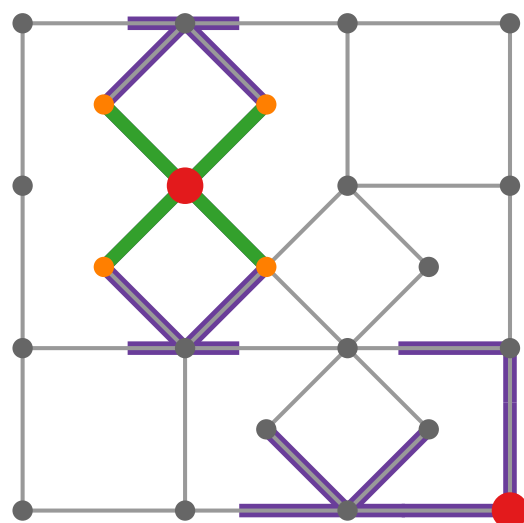
(0) Ball Growth



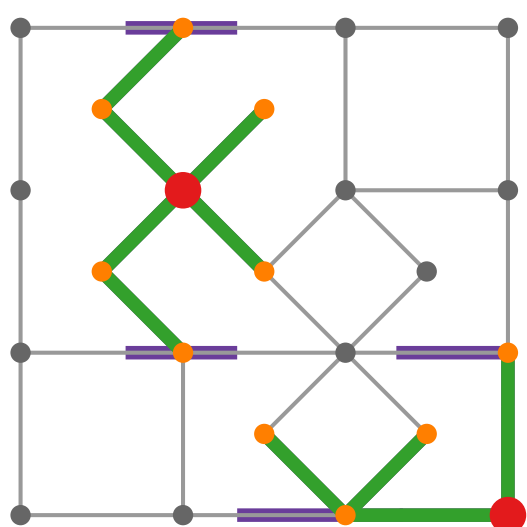
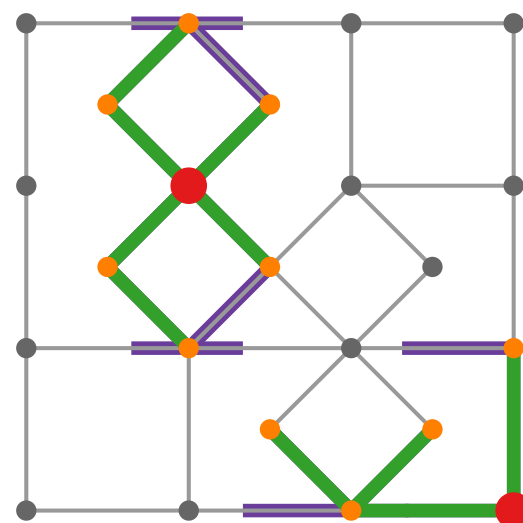
(0) SSSP Cover



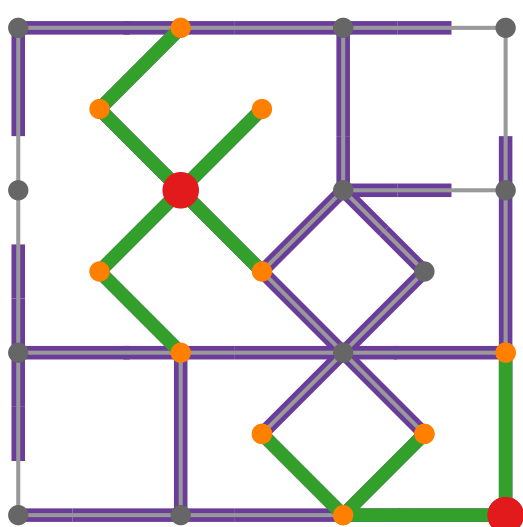
(1) Ball Growth



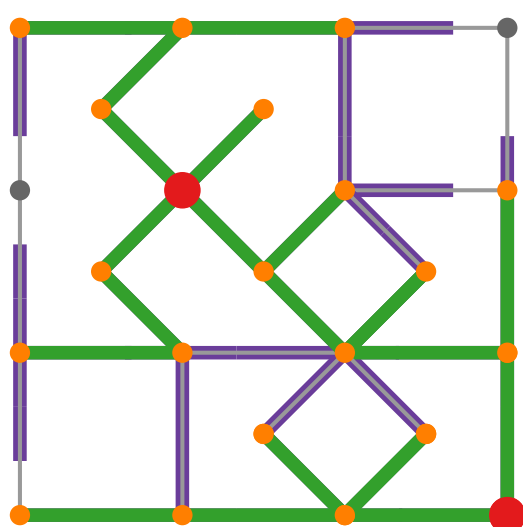
(1) SSSP Cover



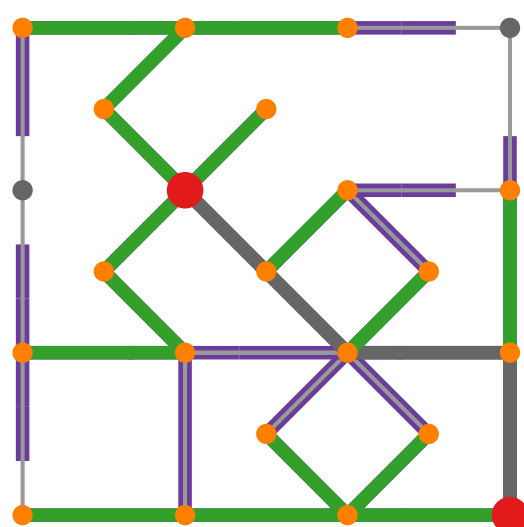
(1) Edge Deletion



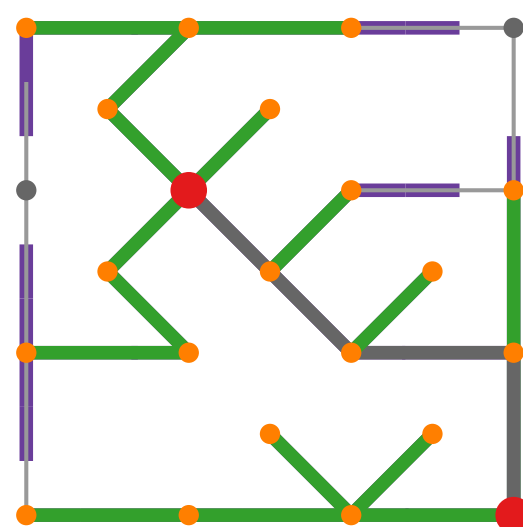
(2) Ball Growth



(2) SSSP Cover



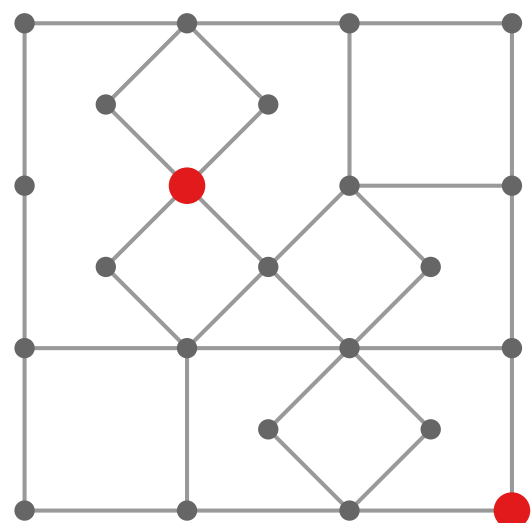
(2) Merge Execution



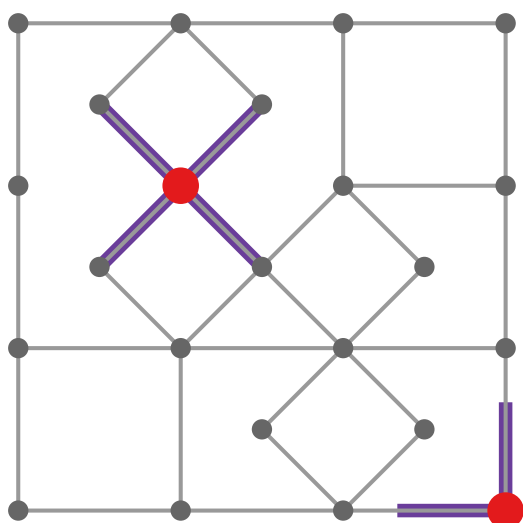
(2) Edge Deletion

The Shell-Decomposition Algorithm: Building Blocks

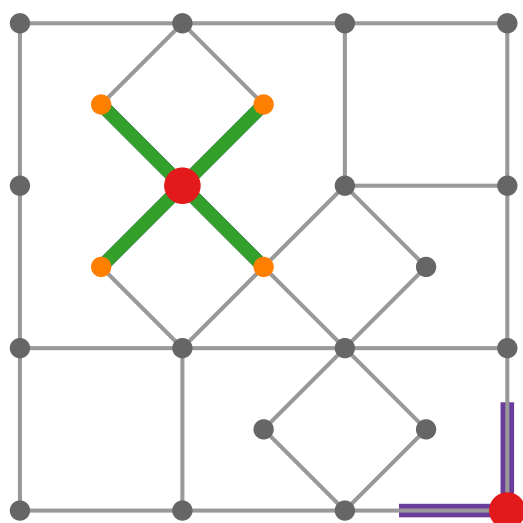
Initialization



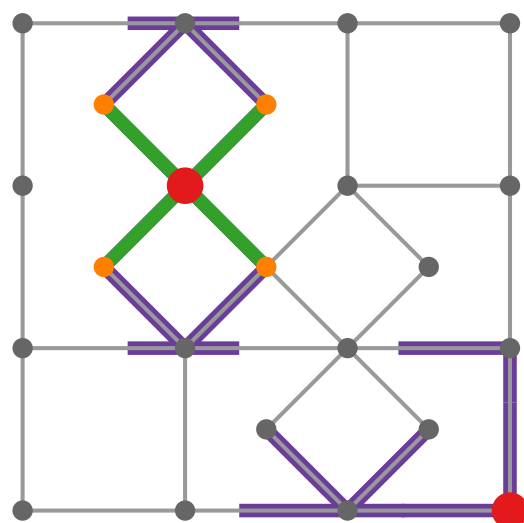
(0) Ball Growth



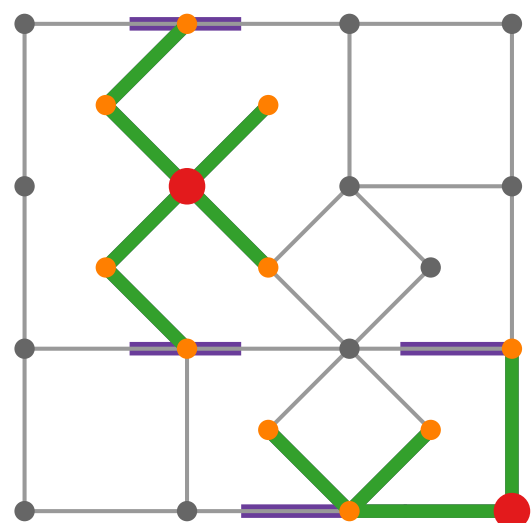
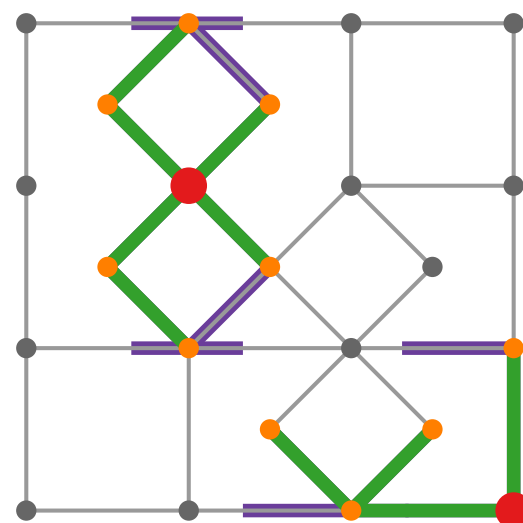
(0) SSSP Cover



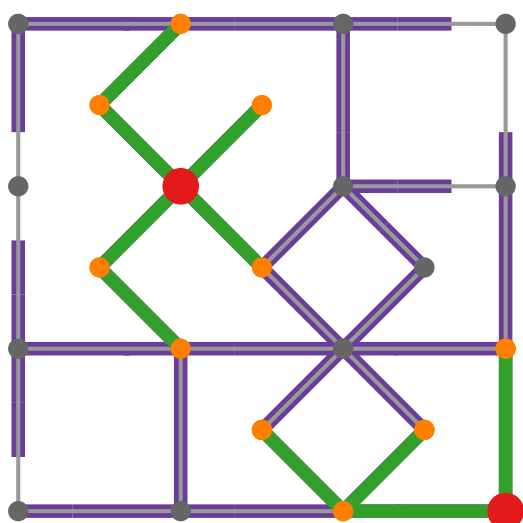
(1) Ball Growth



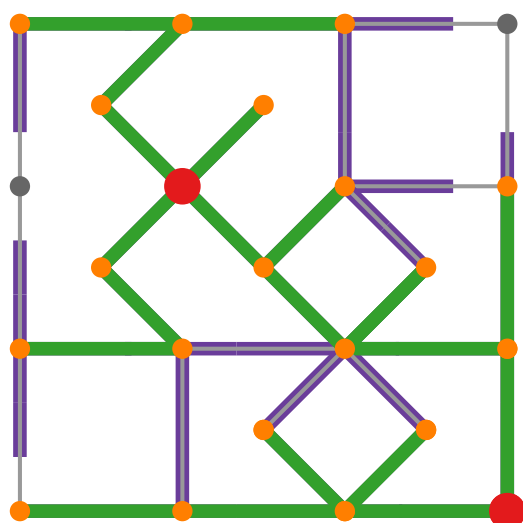
(1) SSSP Cover



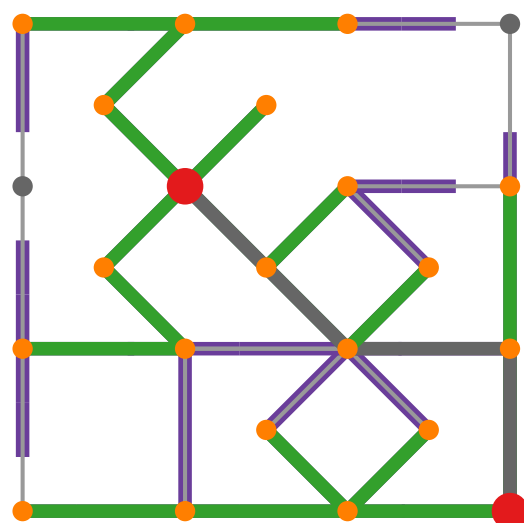
(1) Edge Deletion



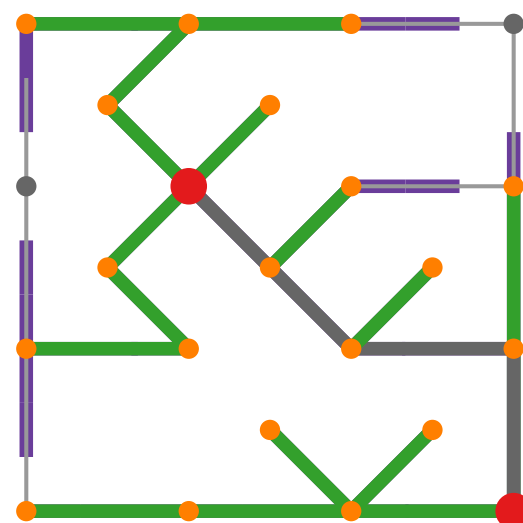
(2) Ball Growth



(2) SSSP Cover

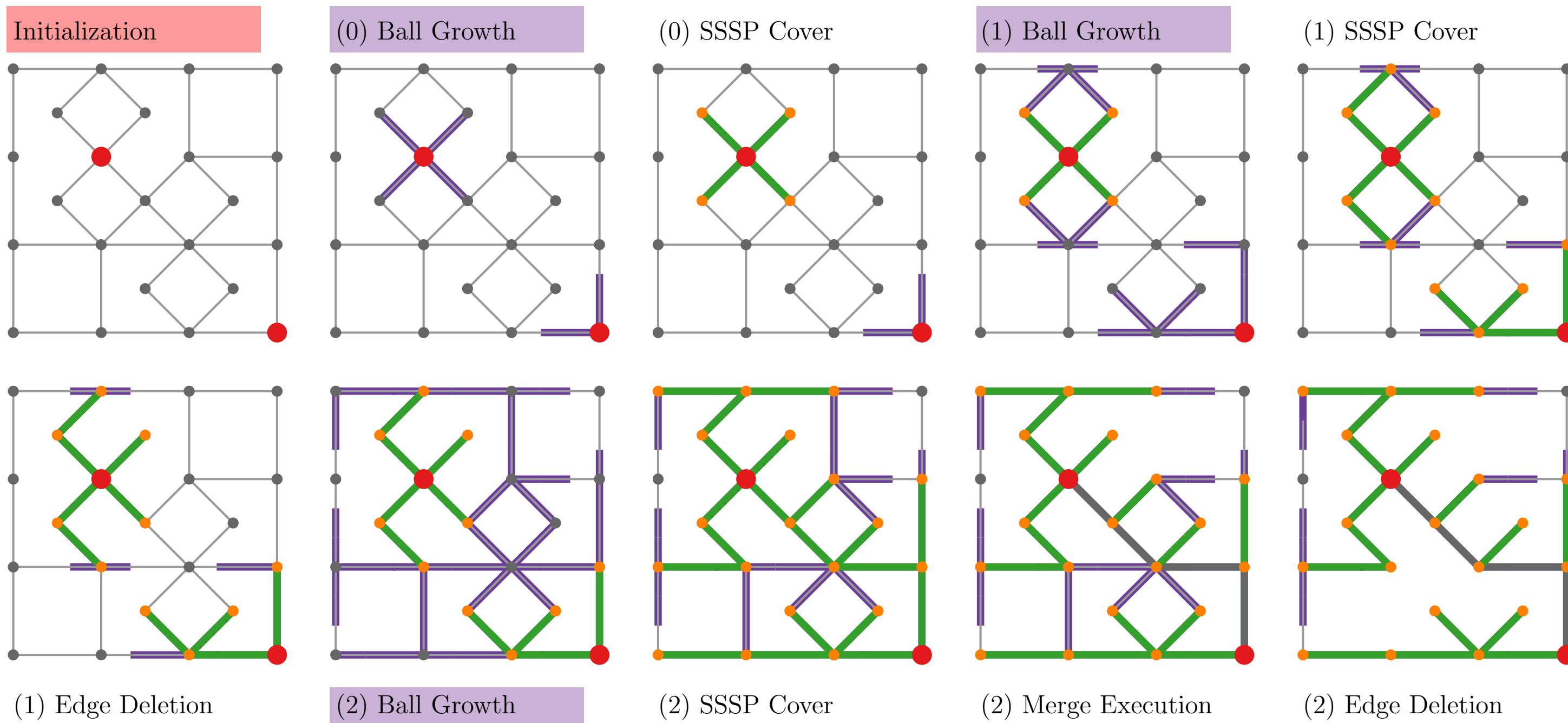


(2) Merge Execution

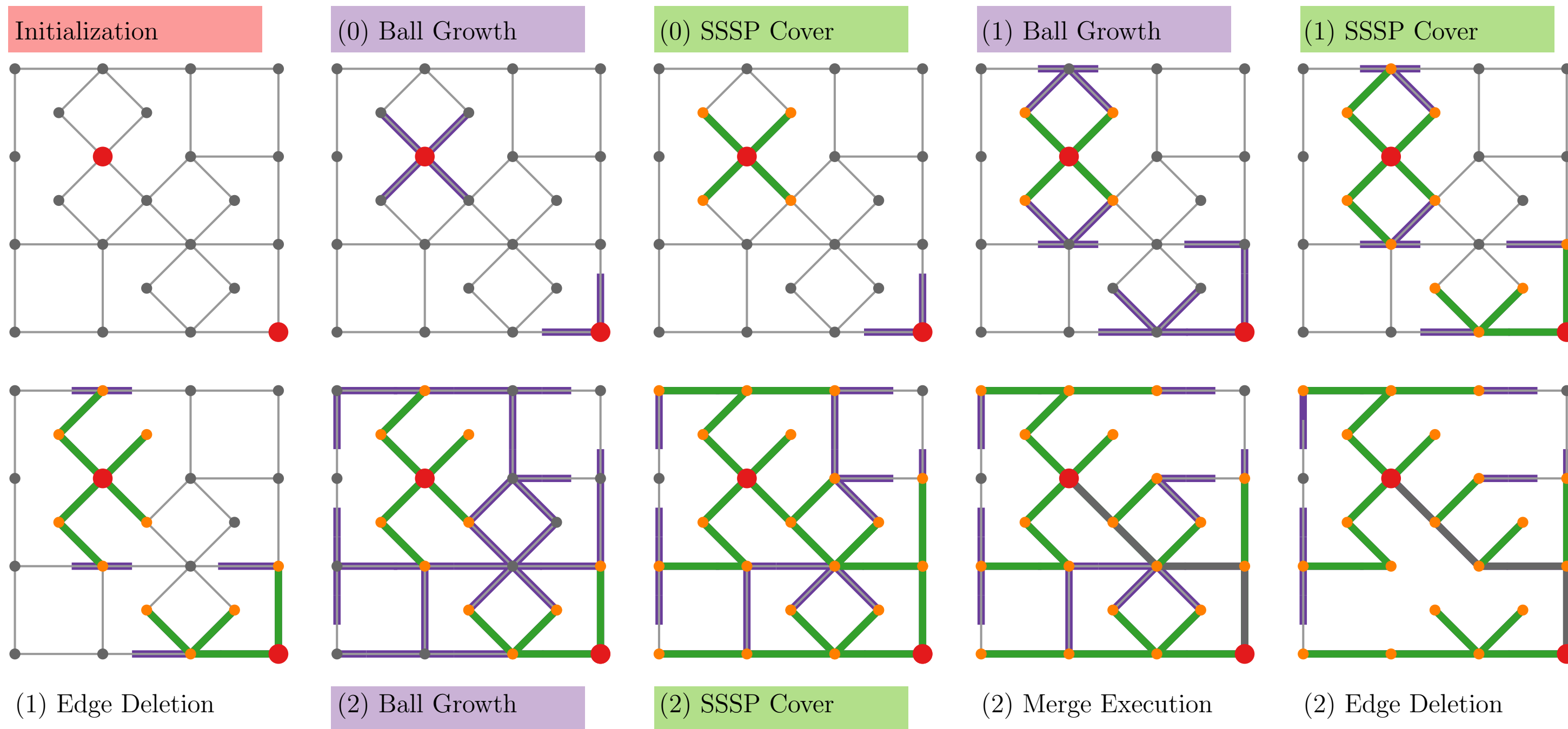


(2) Edge Deletion

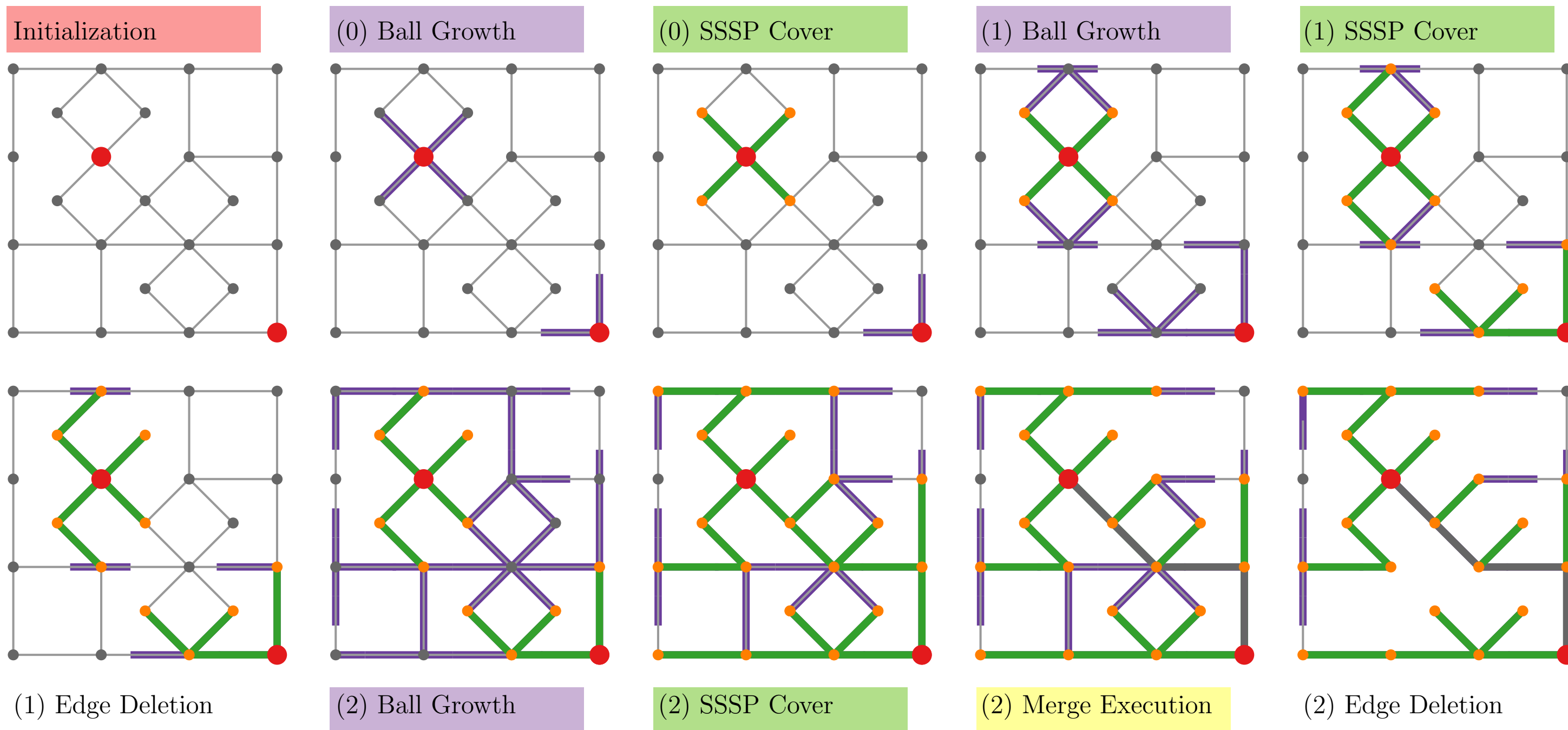
The Shell-Decomposition Algorithm: Building Blocks



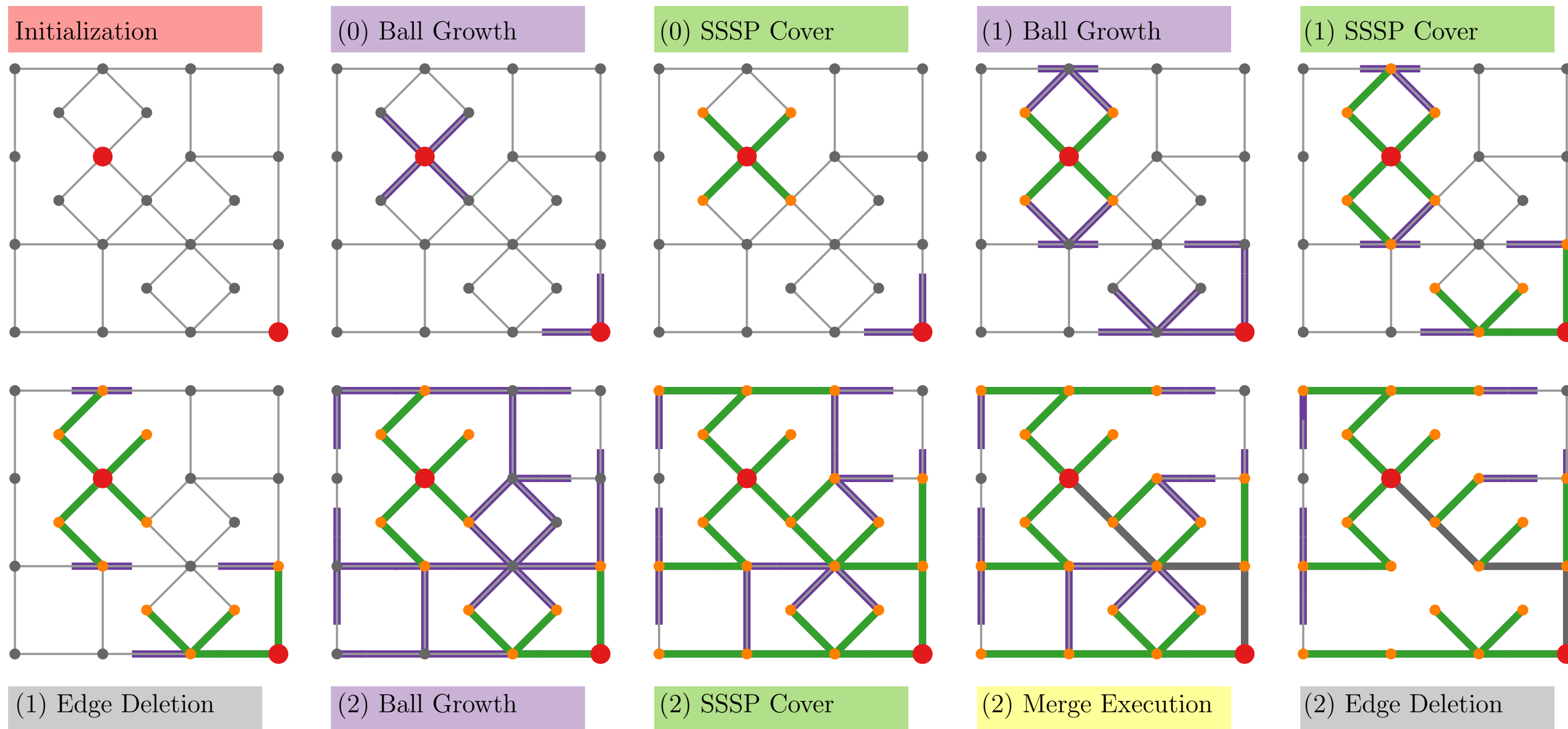
The Shell-Decomposition Algorithm: Building Blocks



The Shell-Decomposition Algorithm: Building Blocks

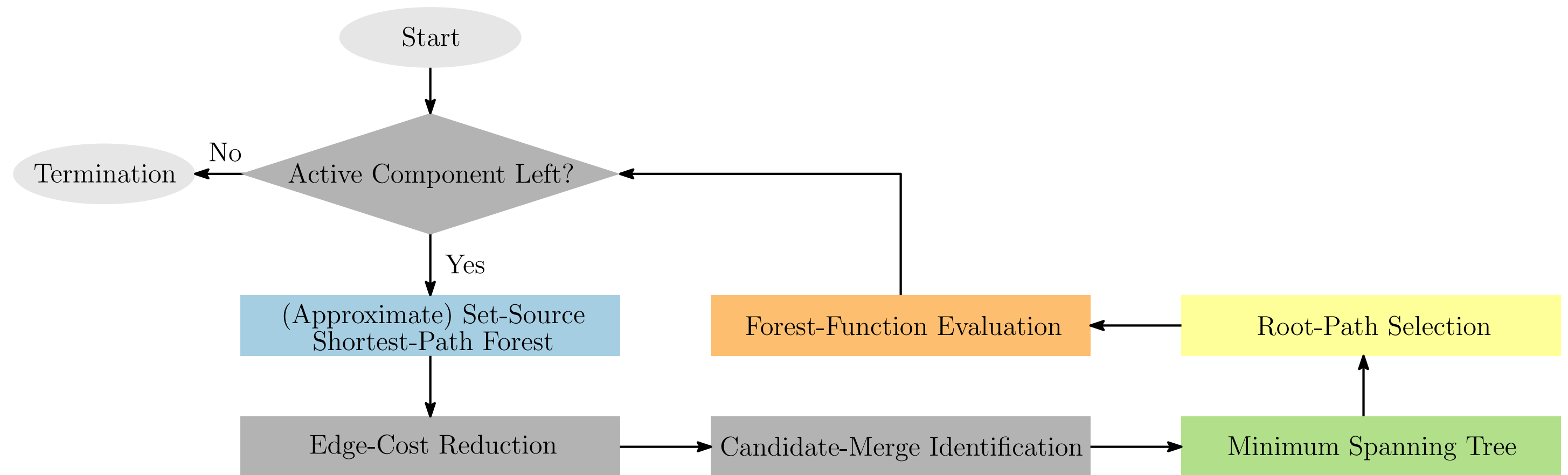


The Shell-Decomposition Algorithm: Building Blocks



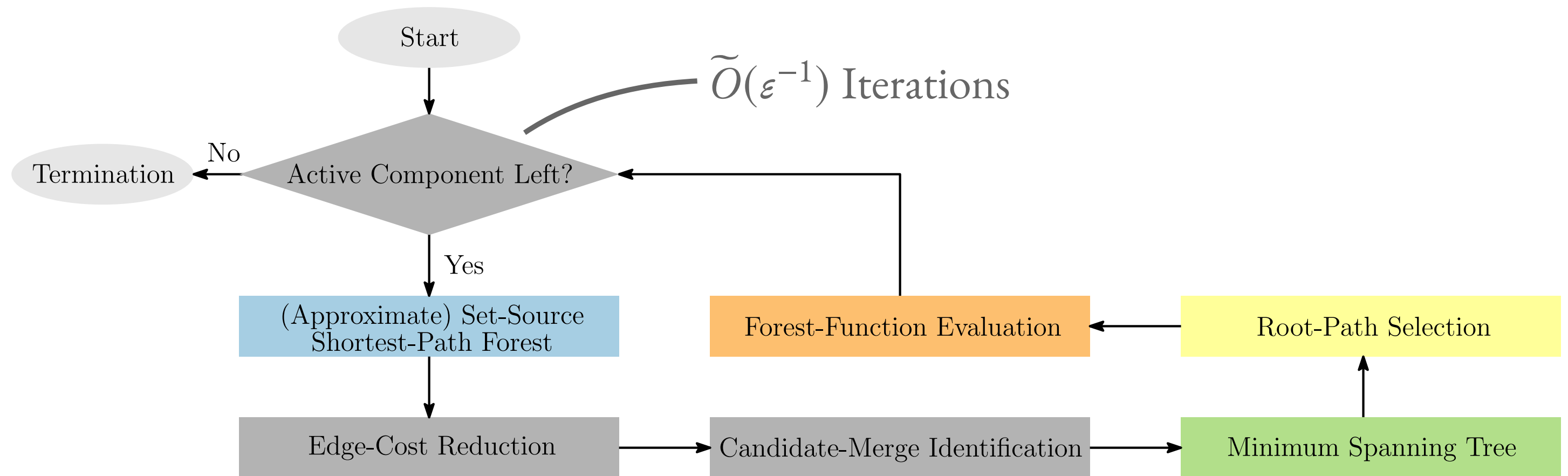
Model-Agnostic Specification

Approximation Guarantee: $2 + \varepsilon$



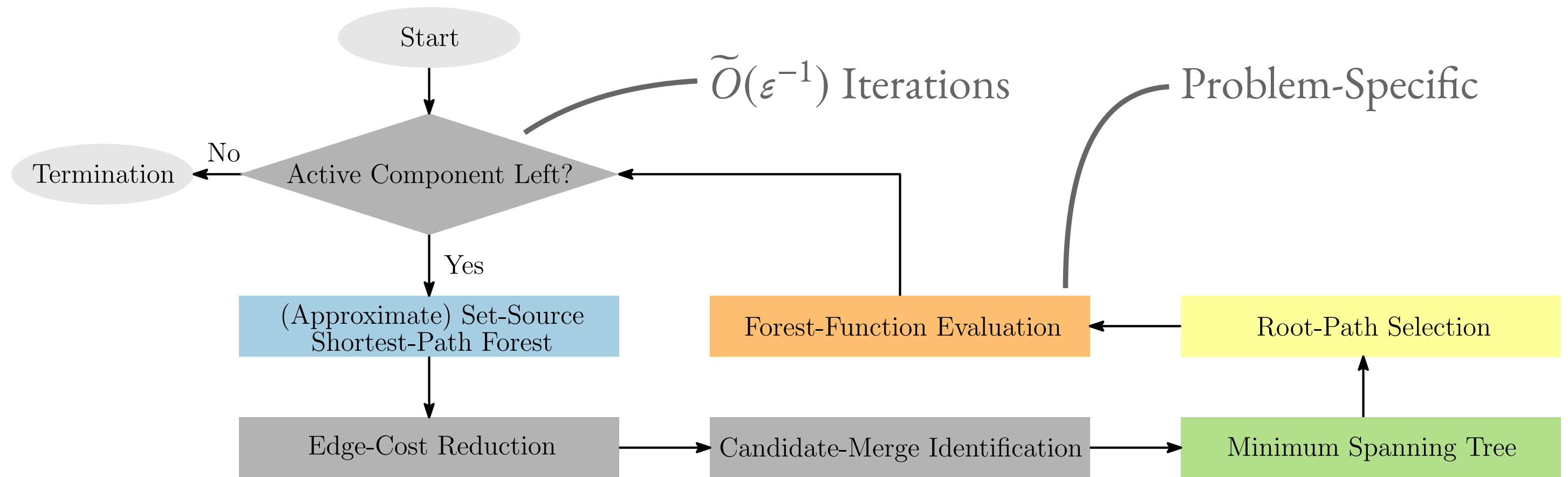
Model-Agnostic Specification

Approximation Guarantee: $2 + \varepsilon$



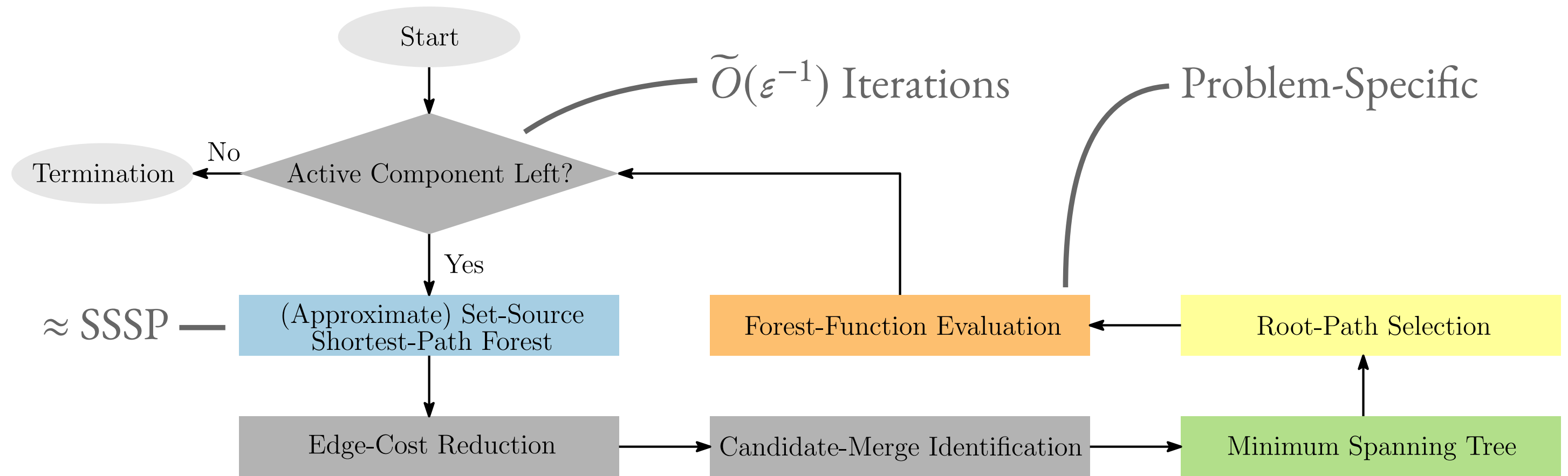
Model-Agnostic Specification

Approximation Guarantee: $2 + \varepsilon$



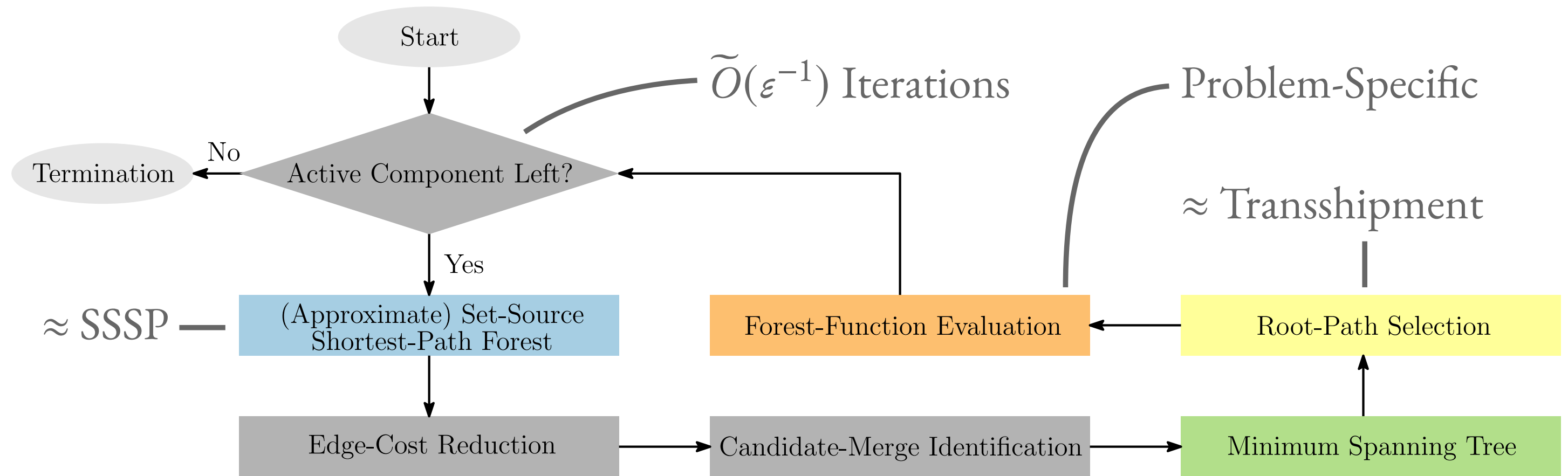
Model-Agnostic Specification

Approximation Guarantee: $2 + \varepsilon$



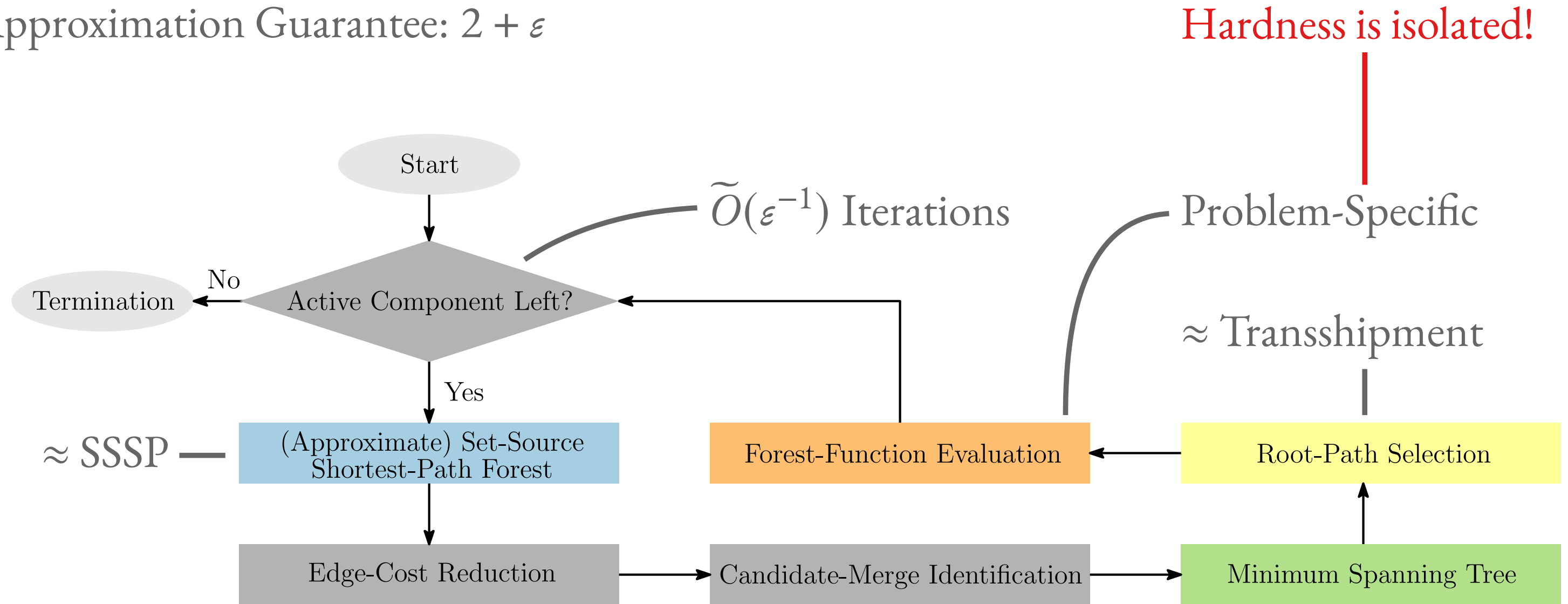
Model-Agnostic Specification

Approximation Guarantee: $2 + \varepsilon$



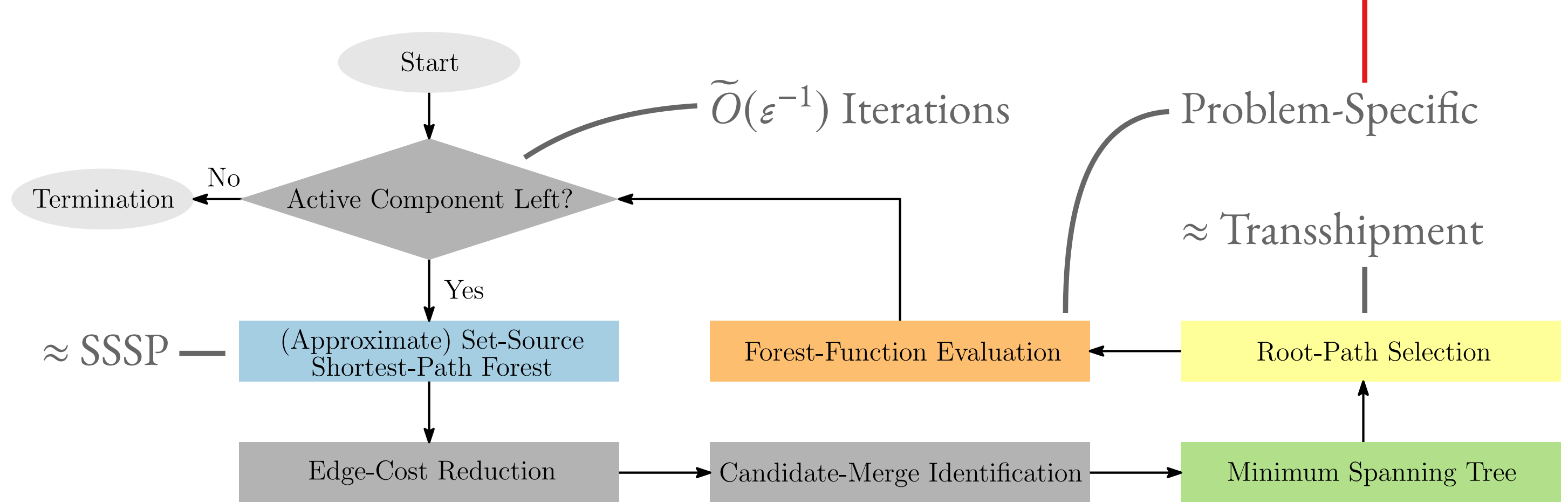
Model-Agnostic Specification

Approximation Guarantee: $2 + \varepsilon$



Model-Agnostic Specification

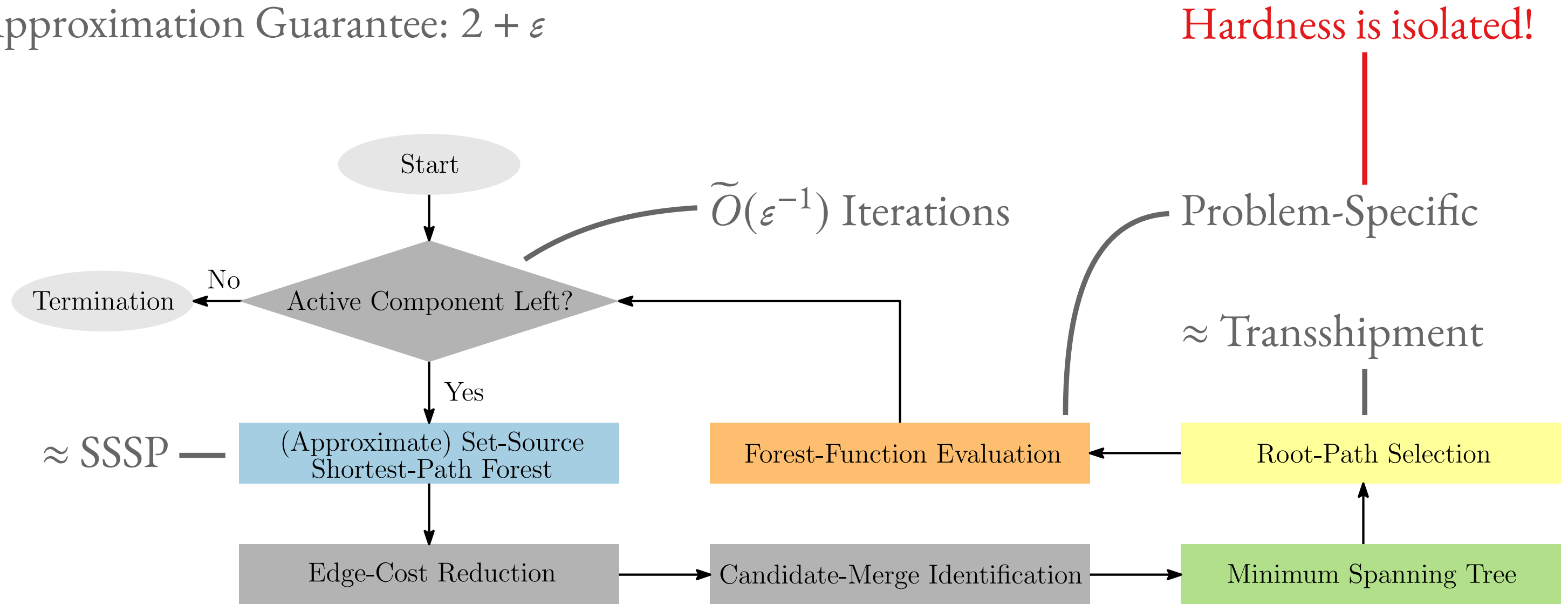
Approximation Guarantee: $2 + \varepsilon$



Complexity: $\tilde{O} \left((\text{aSSSP} + \text{MST} + \text{RPS} + \text{FFE}) \varepsilon^{-1} \right)$

Model-Agnostic Specification

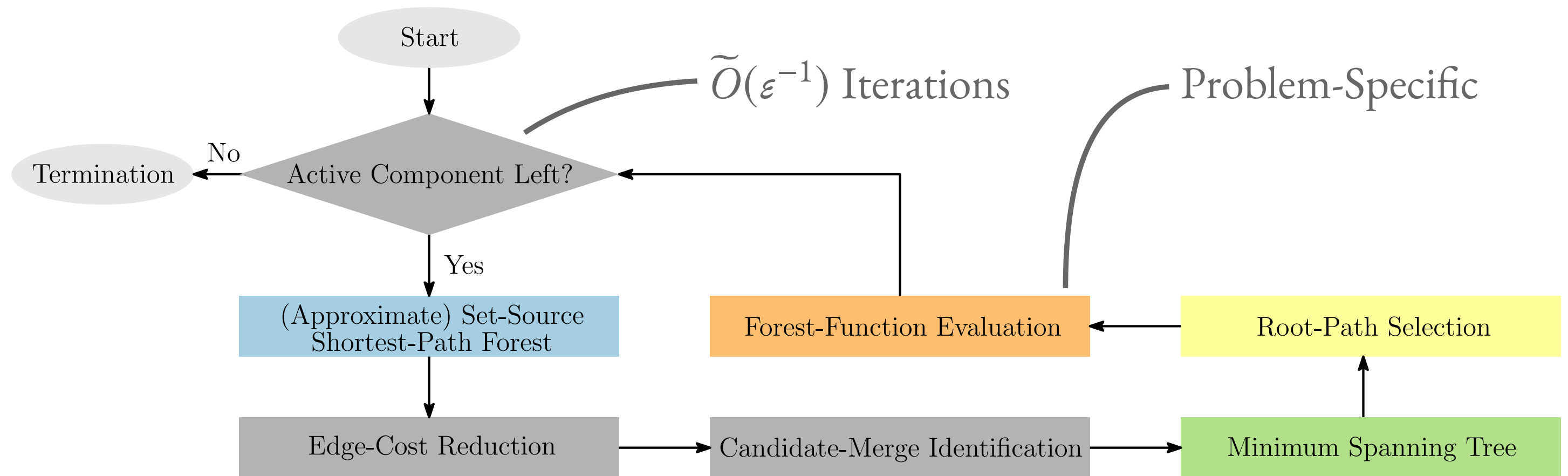
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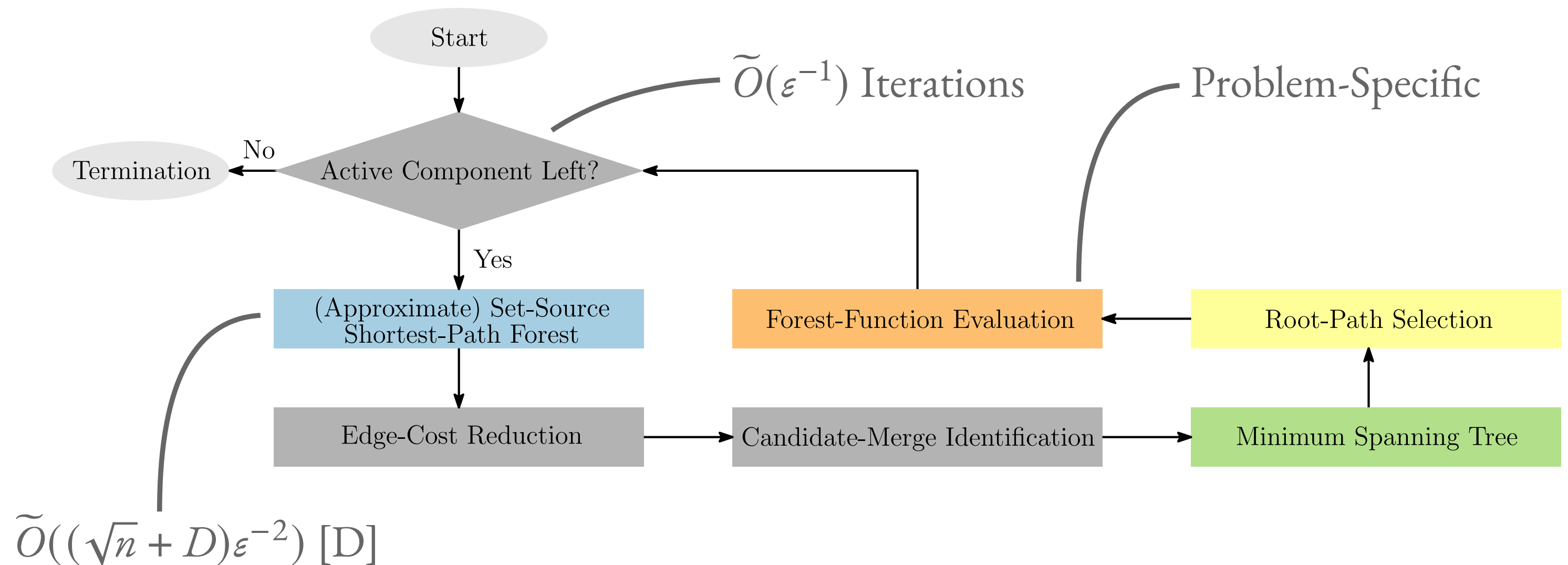
Model-Specific Implementation: CONGEST

$(2 + \varepsilon)$ -approximation with...



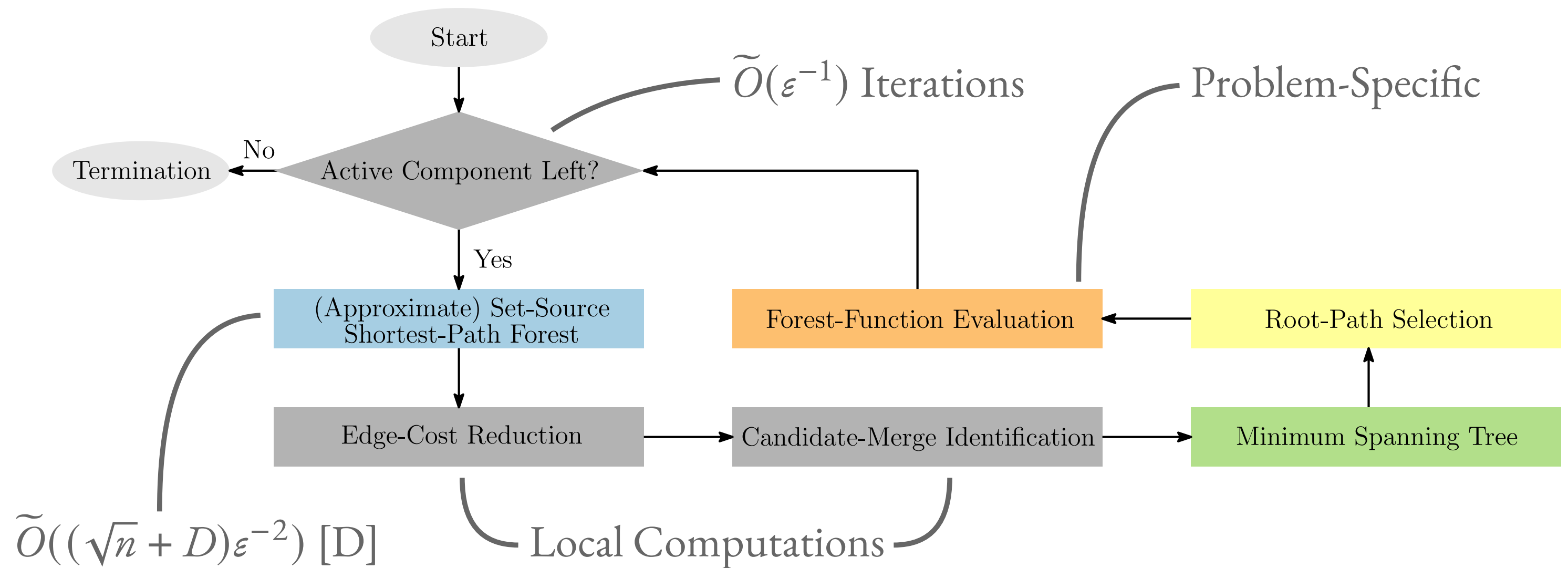
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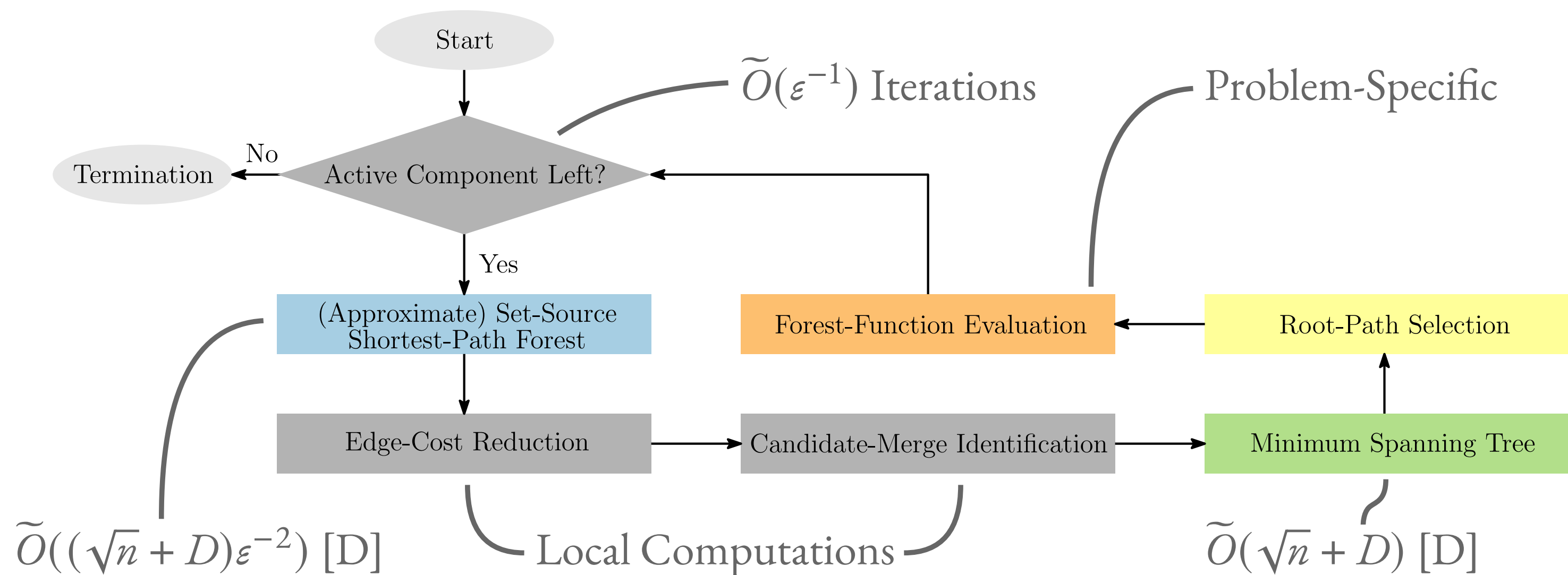
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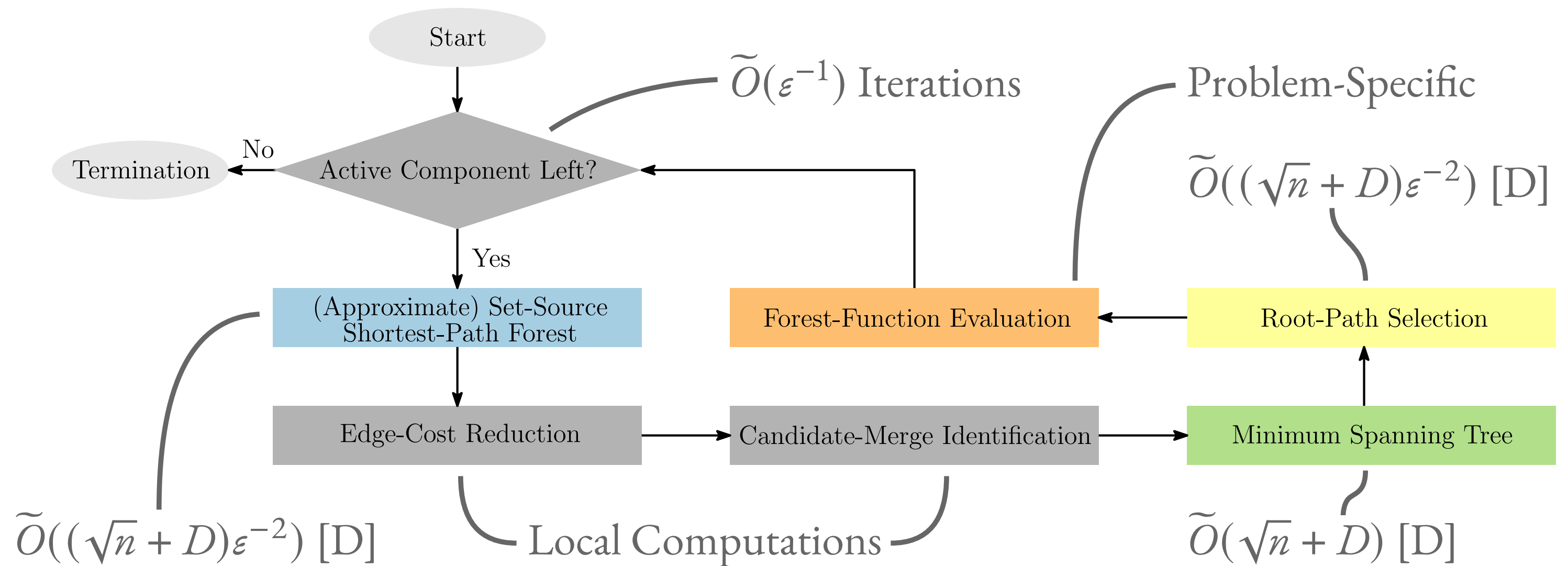
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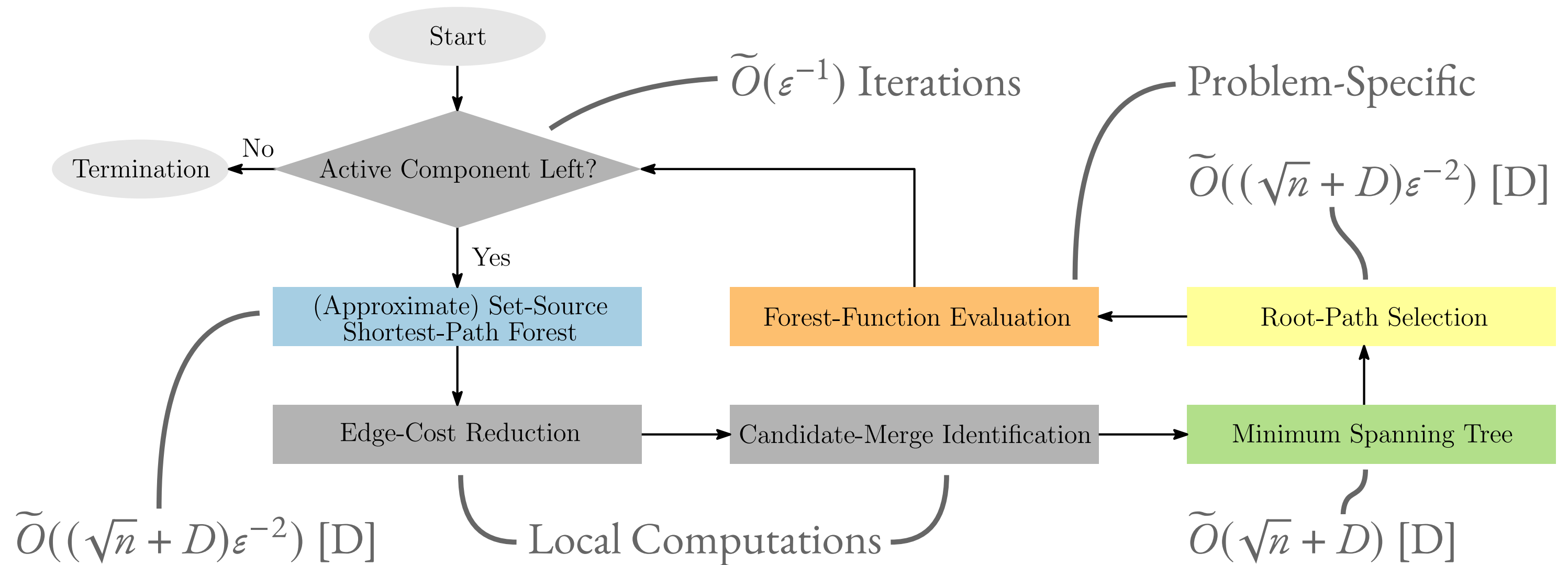
$(2 + \varepsilon)$ -approximation with...



Model-Specific Implementation: CONGEST

$(2 + \varepsilon)$ -approximation with...

Toward universal optimality,
we can replace $\sqrt{n} + D$ by $T^{PA} n^{o(1)}$



Problem-Specific Implementation: Steiner Forest in CONGEST

Interestingly, CONGEST complexity differs depending on the SF input representation!

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Problem	Input	LB	APX	Complexity
SF-IC	Component identifiers $\lambda: V \rightarrow [k] \cup \{\perp\}$; node v knows λ_v	$\tilde{\Omega}(Q + k) R$	$(2 + \varepsilon) D$	$\tilde{O}(\min\{T^{PA} n^{o(1)}, \sqrt{n} + D\} + k)$

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SF-CIC	As in SF-IC, but node v knows λ_v and $ \{u \in V \mid \lambda_u = \lambda_v\} $	$\tilde{\Omega}(Q + k) D$	$(2 + \varepsilon) R$	$\tilde{O}(n^{2/3} + D)$

Problem-Specific Implementation: Steiner Forest in CONGEST

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SF-CR	Each node v is given $\mathcal{R}_v \subseteq V \setminus \{v\}$	$\tilde{\Omega}(Q + t)$ R	$(2 + \varepsilon)$ D	$\tilde{O}(\min\{T^{PA} n^{o(1)}, \sqrt{n} + D\} + t)$

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SF-SCR	$\mathcal{R} \subseteq \binom{V}{2}$; node v knows $\mathcal{R}_v = \{u \in V \mid \{u, v\} \in \mathcal{R}\}$	$\tilde{\Omega}(Q+t)$ D	$(2+\varepsilon)$ R	$\tilde{O}(\min\{T^{PA}n^{o(1)}, \sqrt{n}+D\})$

Outlook

Outlook

Our Main Contribution: Shell-Decomposition Algorithm

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General Framework for Model-Agnostic Approximation of CFPs

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Instantiated in 3 Models for 3 Problems, improving SOTA esp. for SF in CONGEST

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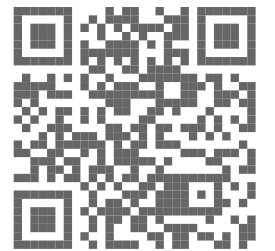
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Thank You!



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