Fermionic Advantage in Distributed Computing

Fatemeh Moradi Kalarde
Inria de Saclay, PhIQuS, 22.10.2025

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Qubits
$\cup$ $\}$ centralised/distributed
Classical bits

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$$\left.\begin{array}{c}\text{Febits}\\ \cup \\ \text{Qubits}\end{array}\right\}\ \text{distributed}$$

$$\left.\begin{array}{c}\text{Qubits}\\ \cup \\ \text{Classical bits}\end{array}\right\}\ \text{centralised/distributed}$$

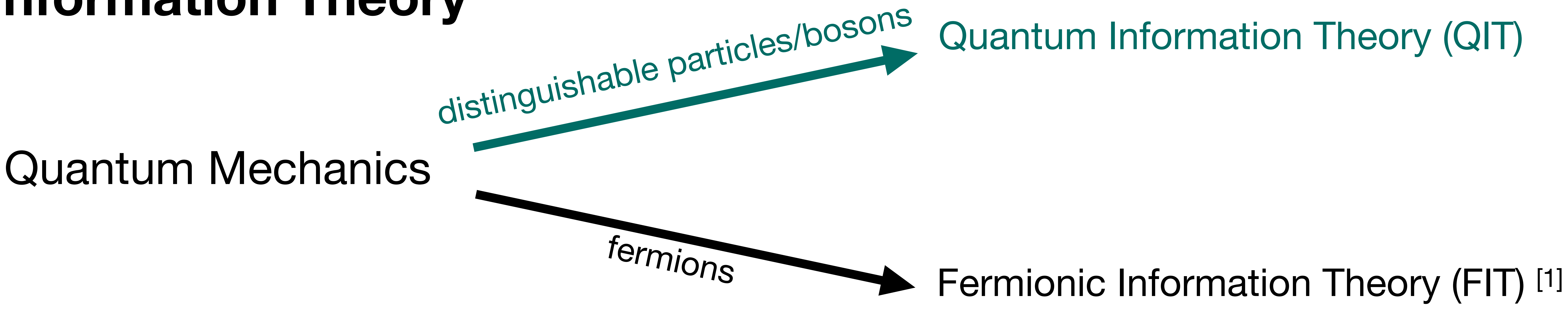
Outline

- Introduction to Fermionic Information Theory (FIT)
- Review 1: Qubits vs. Classical bits: a mathematical comparison
- Febits vs. Qubits: a mathematical comparison
- Review 2: Bell (CHSH) game: Qubits beat Classical bits
- Our game: Febits beat Qubits

Information Theory

Quantum Mechanics $\xrightarrow{\text{distinguishable particles/bosons}}$ Quantum Information Theory (QIT)

Information Theory



[1] Nicetu Tibau Vidal, Mohit Lal Bera, Arnau Riera, Maciej Lewenstein, and Manabendra Nath Bera. Quantum operations in an information theory for fermions. Phys. Rev. A, 104:032411, Sep 2021.

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Quantum Mechanics $\xrightarrow{\text{fermions}}$ Fermionic Information Theory (FIT) [1]

1. Wedge product: $$|1\rangle_1 \wedge |1\rangle_2 = -\,|1\rangle_2 \wedge |1\rangle_1$$

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$$\rho = \rho_e \oplus \rho_o = \begin{pmatrix} \rho_e & 0 \\ 0 & \rho_o \end{pmatrix} \qquad \hat{O} = \hat{O}_e \oplus \hat{O}_o = \begin{pmatrix} \hat{O}_e & 0 \\ 0 & \hat{O}_o \end{pmatrix}$$

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Review 1: Qubits vs. Classical bits: Mapping

Classical bits $\longrightarrow$ **Qubits**

$$0_c \qquad |0\rangle_Q$$
$$1_c \qquad |1\rangle_Q$$

Review 1: Qubits vs. Classical bits: Mapping

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Qubits	$\longrightarrow$	**Classical bits**
1. $\alpha\|0\rangle + \beta\|1\rangle$	$\longrightarrow$	Infinitely many classical bits needed!

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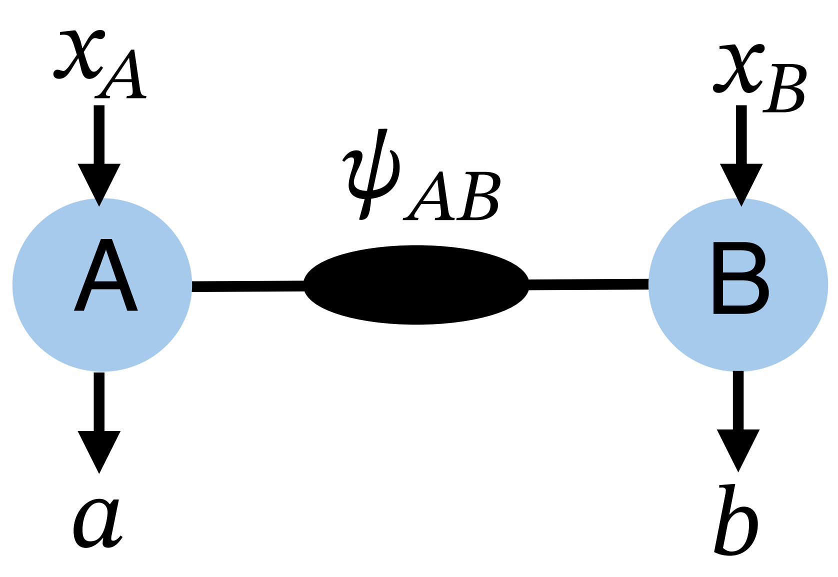
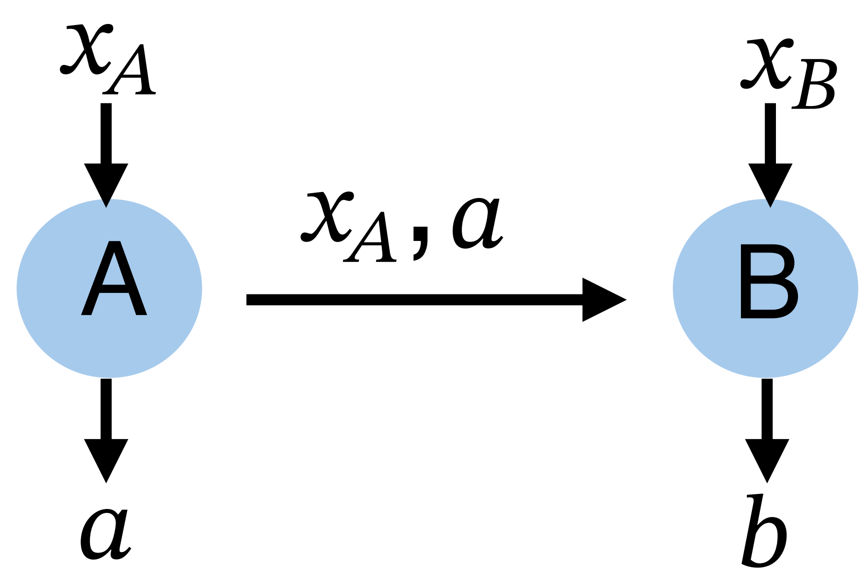
2. EPR:

Local operation $\longrightarrow$ Non-local operation





$$|\psi_{AB}\rangle = \frac{|00\rangle_{AB} + |11\rangle_{AB}}{\sqrt{2}}$$

$a = 0, x_A = 1 \rightarrow |+\rangle_B$

$a = 1, x_A = 1 \rightarrow |-\rangle_B$

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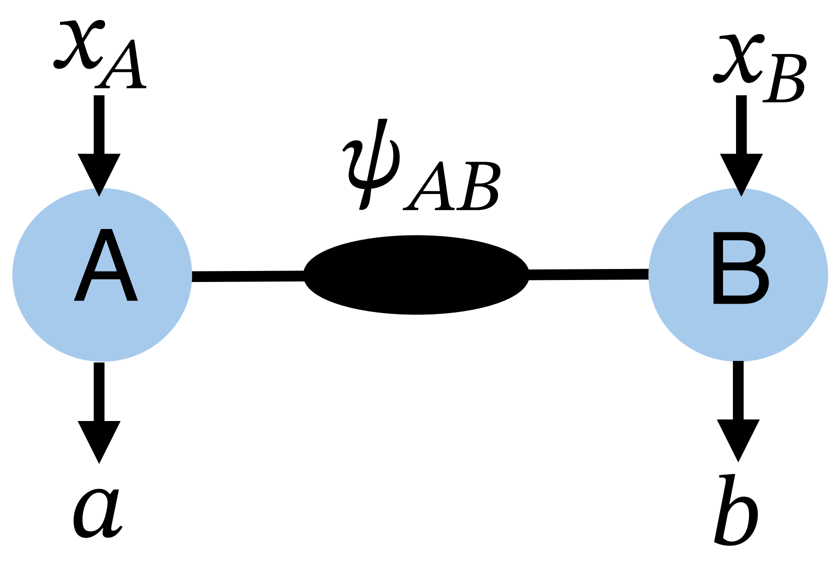

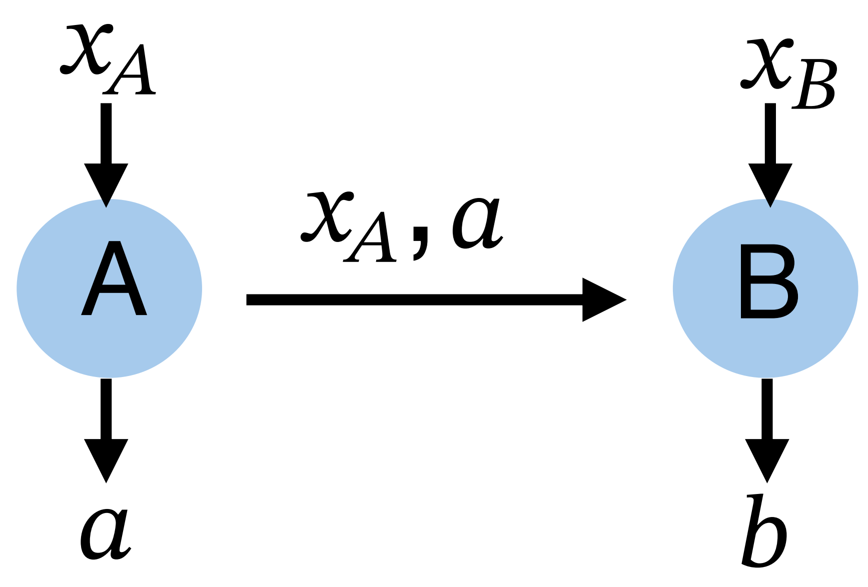

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Ok… but is there a fundamental difference? 🤔

Review 2: CHSH game

Febits vs. Qubits: Mapping

Qubits $\longrightarrow$ **Febits**

$$|0\rangle_Q \rightarrow |0_1 0_2\rangle_F$$

$$|1\rangle_Q \rightarrow |1_1 1_2\rangle_F$$

Febits vs. Qubits: Mapping

Qubits $\longrightarrow$ **Febits**

$$|0\rangle_Q \to |0_1 0_2\rangle_F$$

$$|1\rangle_Q \to |1_1 1_2\rangle_F$$

$$X^{(Q)} \to f_1^\dagger f_2^\dagger + f_2 f_1$$

Febits vs. Qubits: Mapping

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$$X^{(Q)} \rightarrow f_1^\dagger f_2^\dagger + f_2 f_1$$

1 qubit $\longrightarrow$ 2 Febits

- Separable states remain separable
- Operators remain local

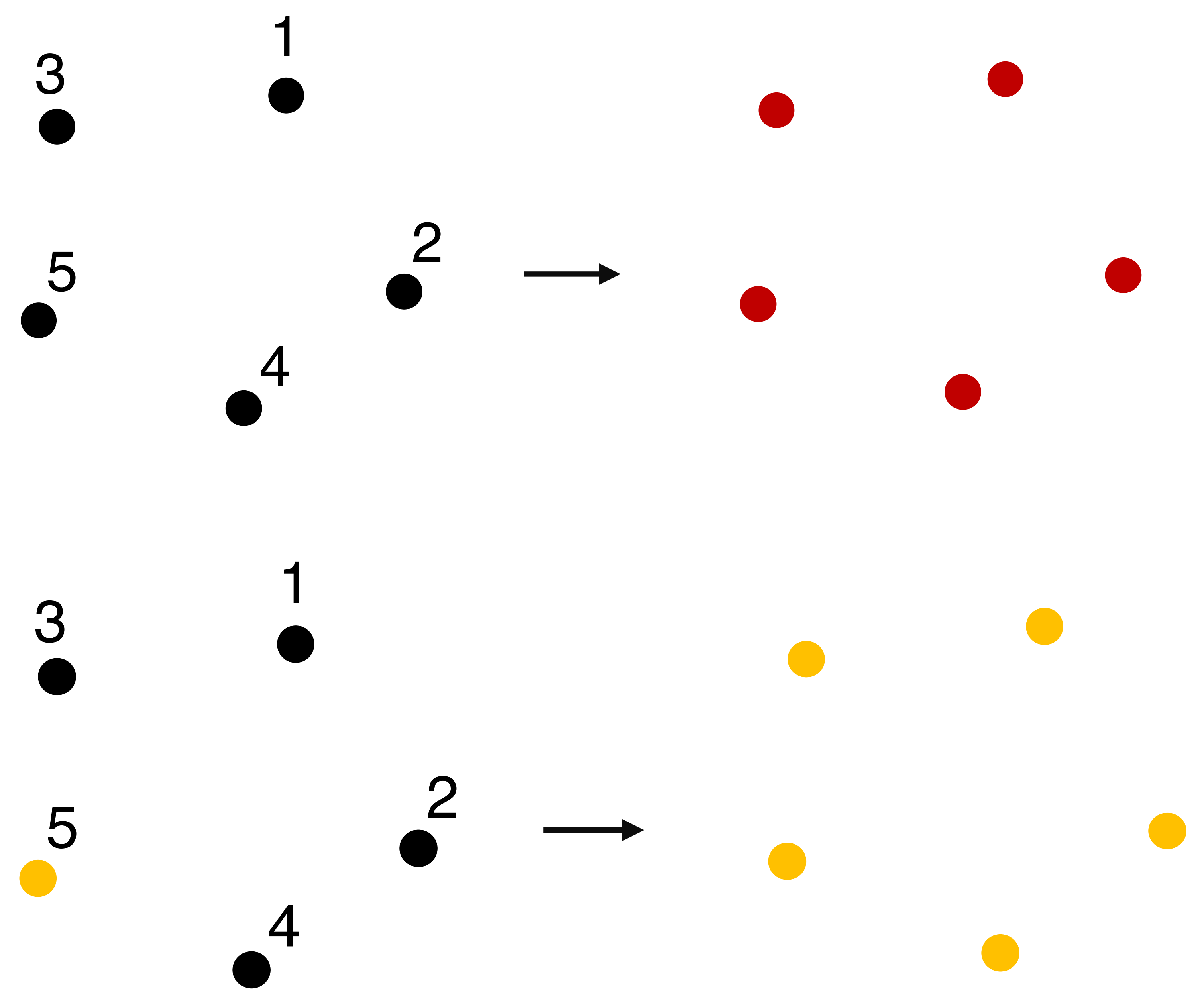
Febits vs. Qubits: Mapping

Febits ⟶ **Qubits**

1. Jordan Wigner:

$$|n_1 n_2 \ldots n_m\rangle_F \longmapsto |n_1 n_2 \ldots n_m\rangle_Q$$

$$X_j^{(F)} \longmapsto \left(\prod_{k=1}^{j-1} Z_k^{(Q)}\right) X_j^{(Q)}$$



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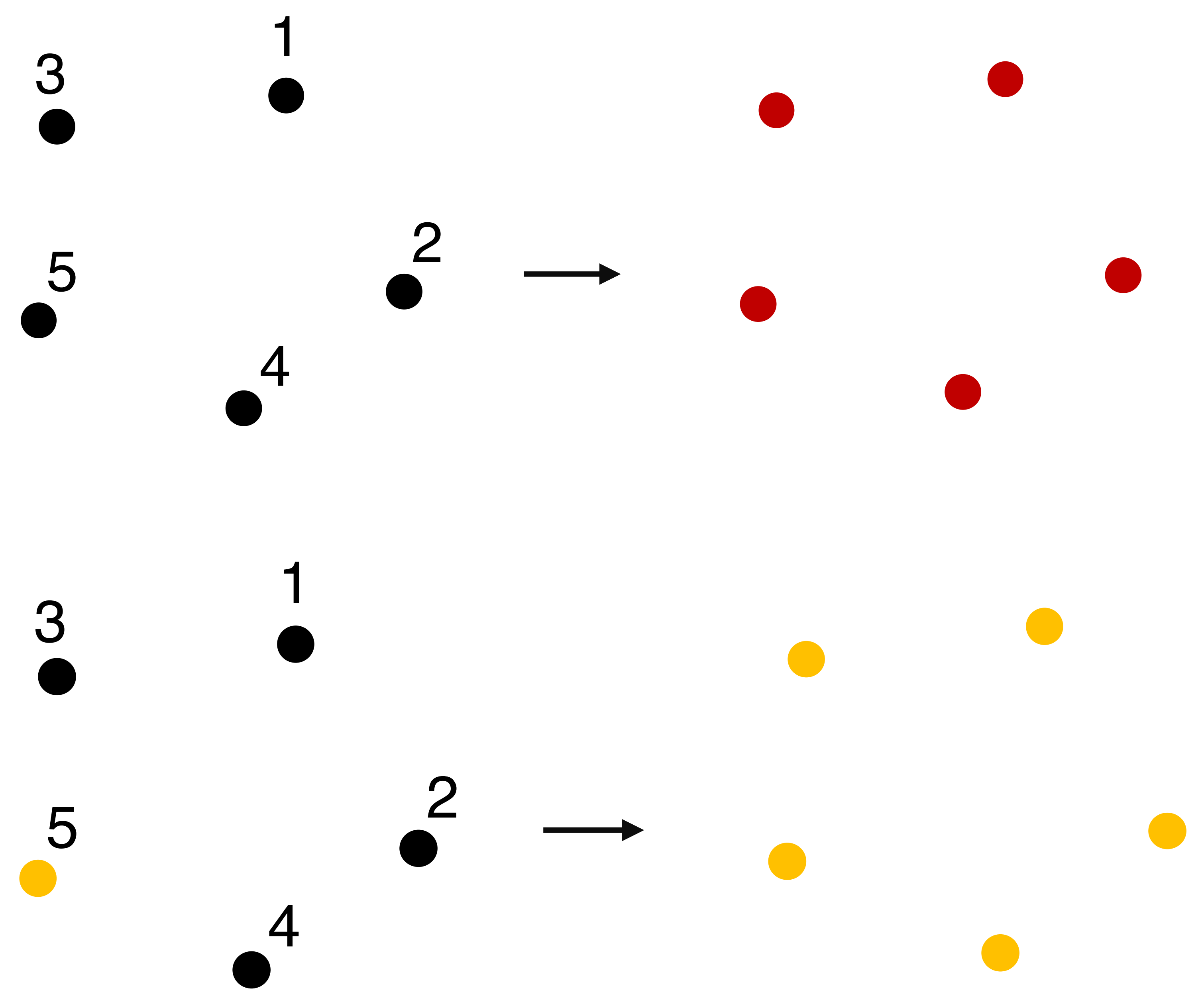
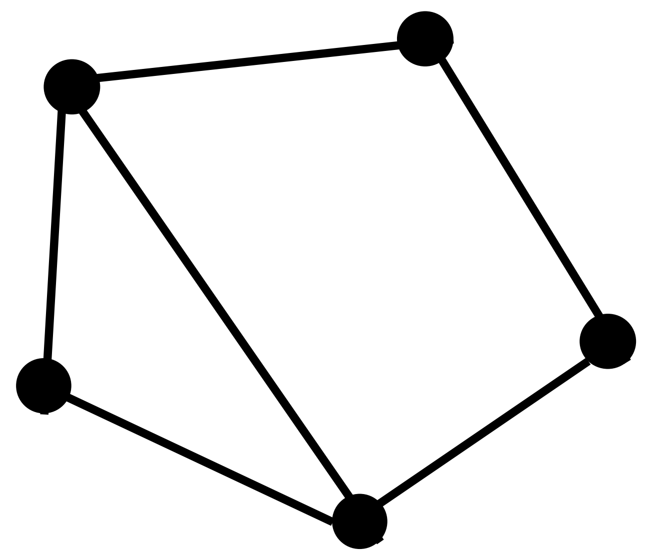
- Separable states remain separable
- Operators become highly nonlocal



Febits vs. Qubits: Mapping

Febits ⟶ **Qubits** ***with a given locality structure***

2. Superfast Bravy Kitaev:



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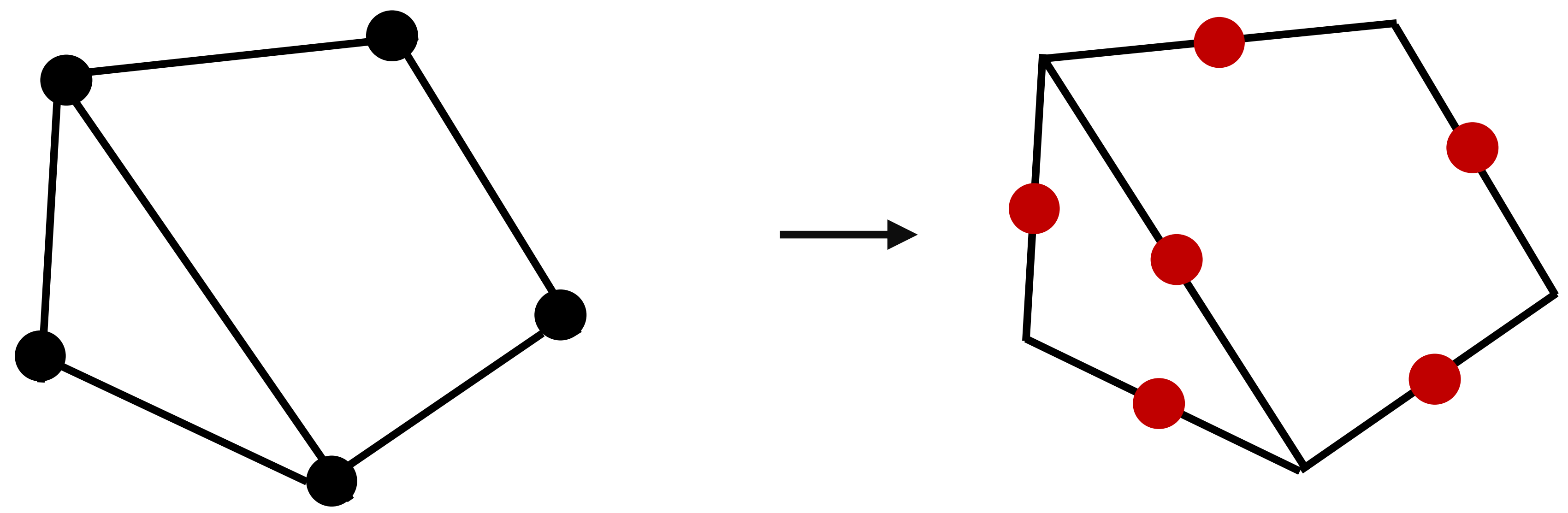

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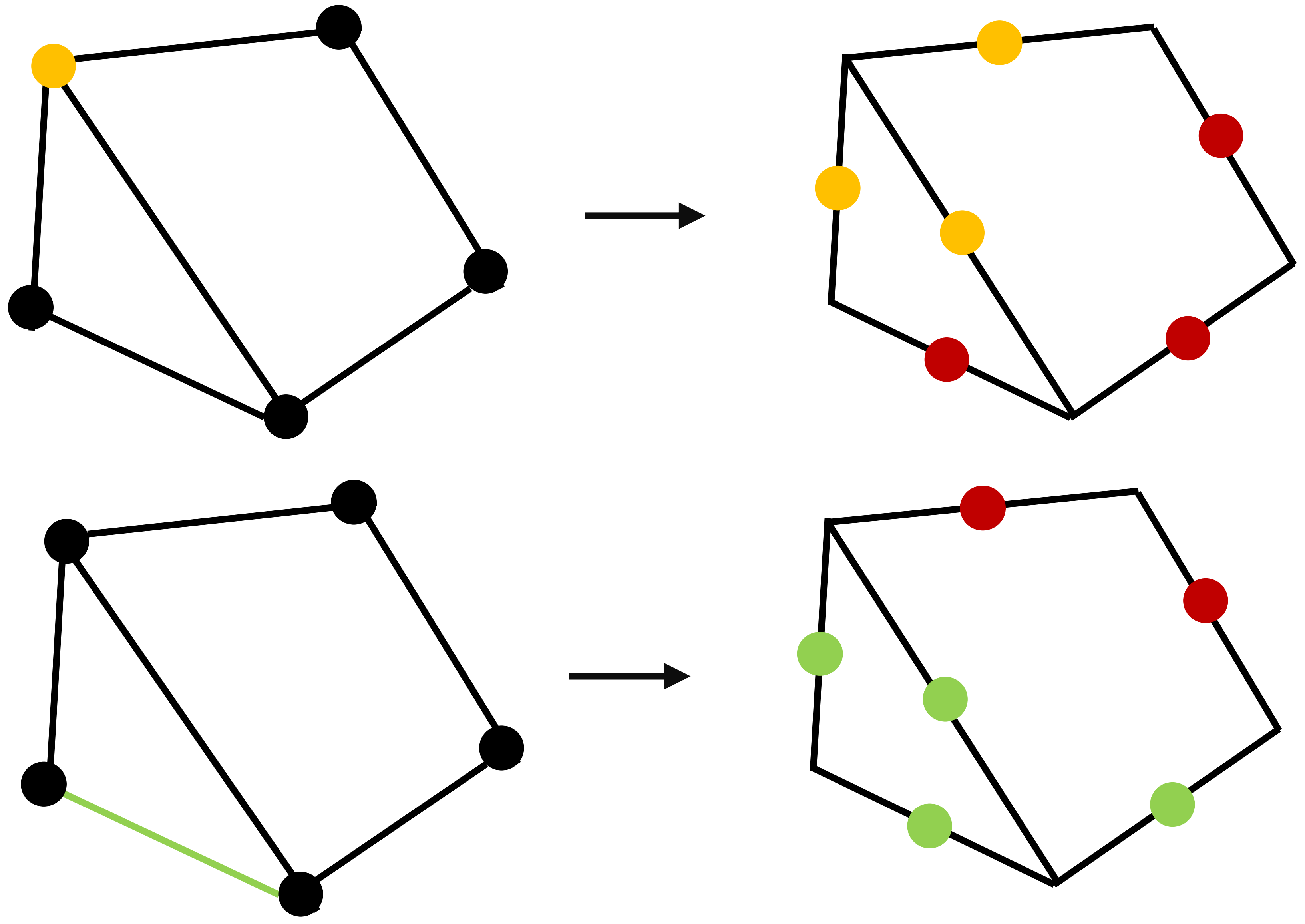

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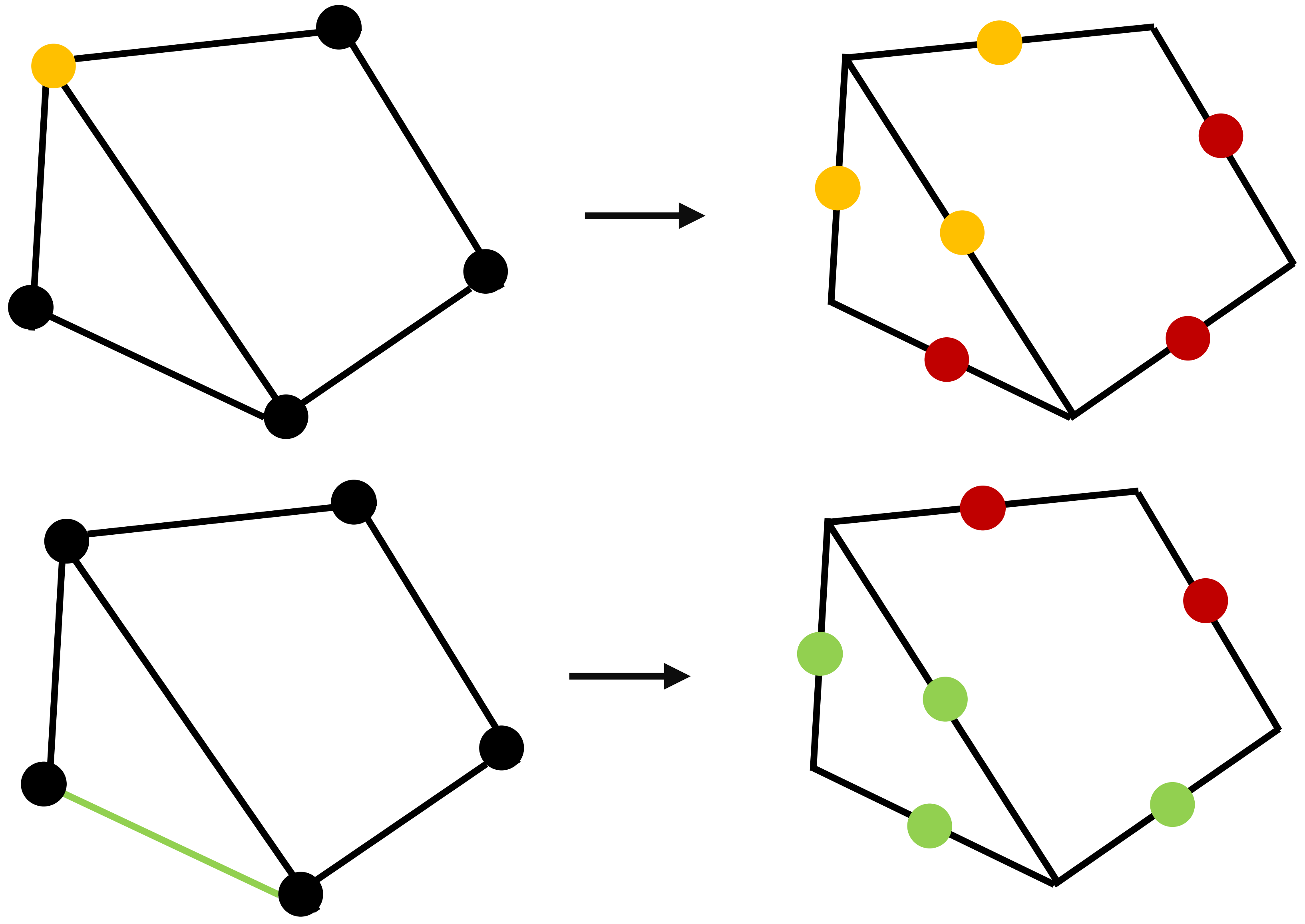

⚠️ Downside: State preparation can be hard ⚠️

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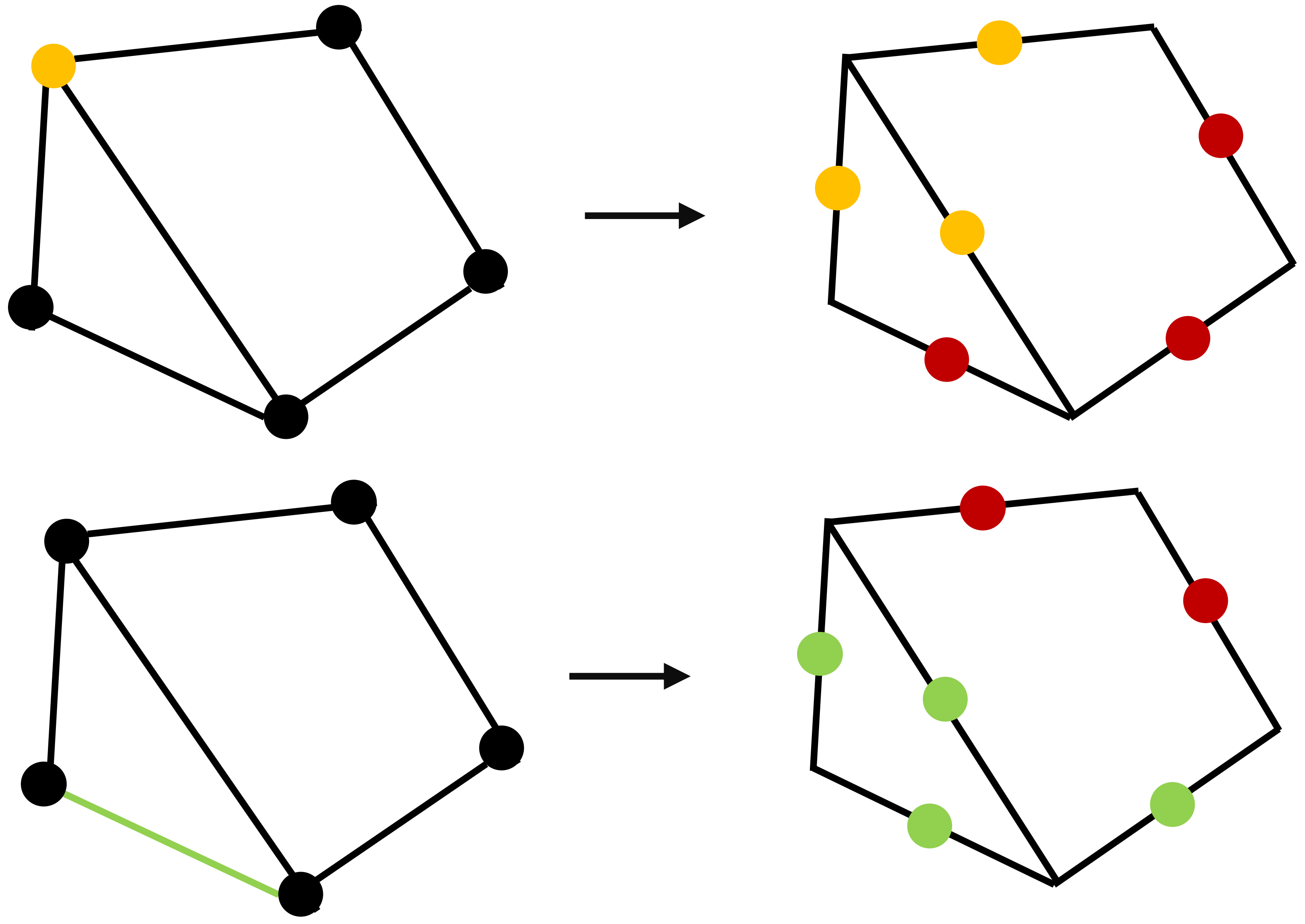

⚠️ Downside: State preparation can be hard ⚠️

- Operators remain roughly local
- Separable states become entangled

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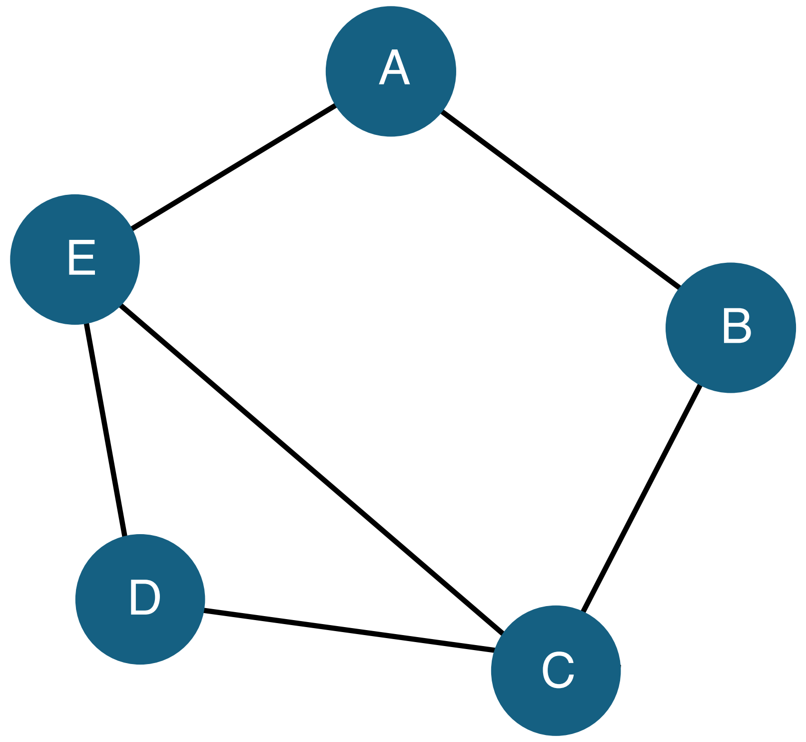
🤔 Ok… but is there a fundamental difference?

Distributed Computing: LOCAL model



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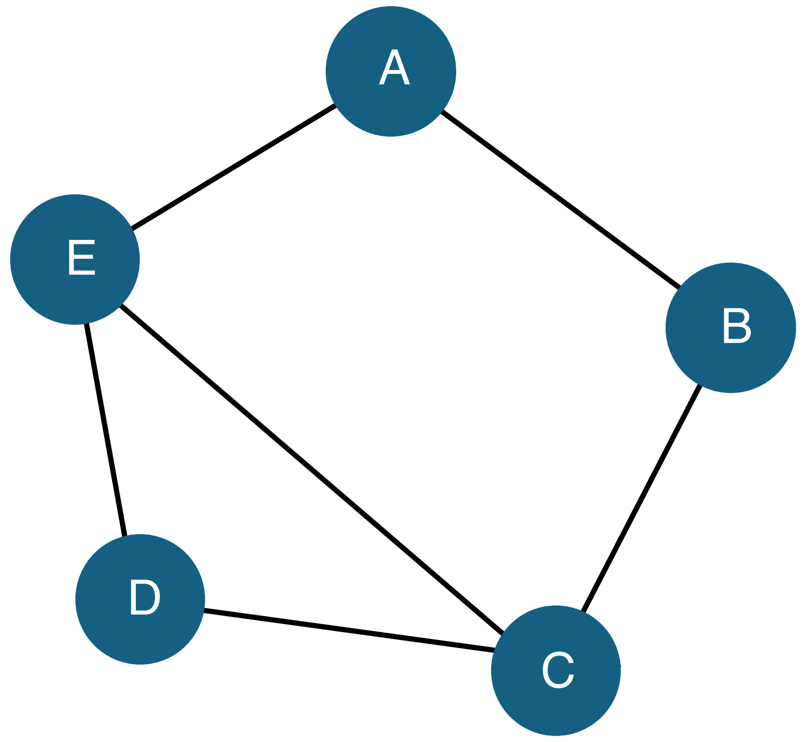
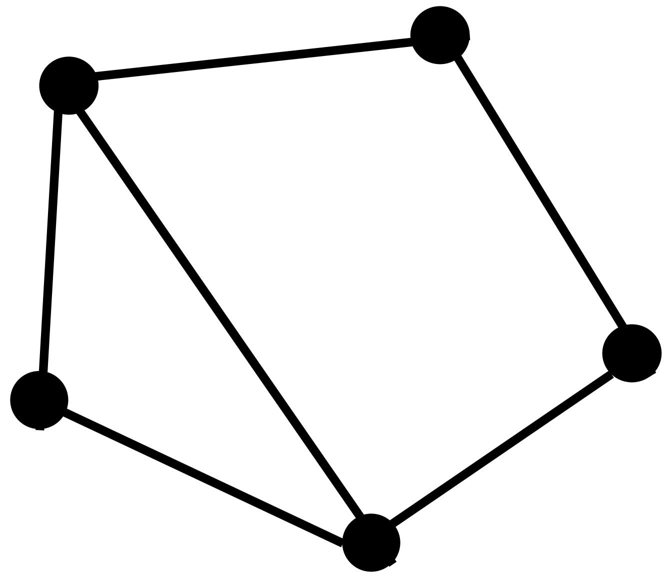


Fermionic to Qubit: SBK



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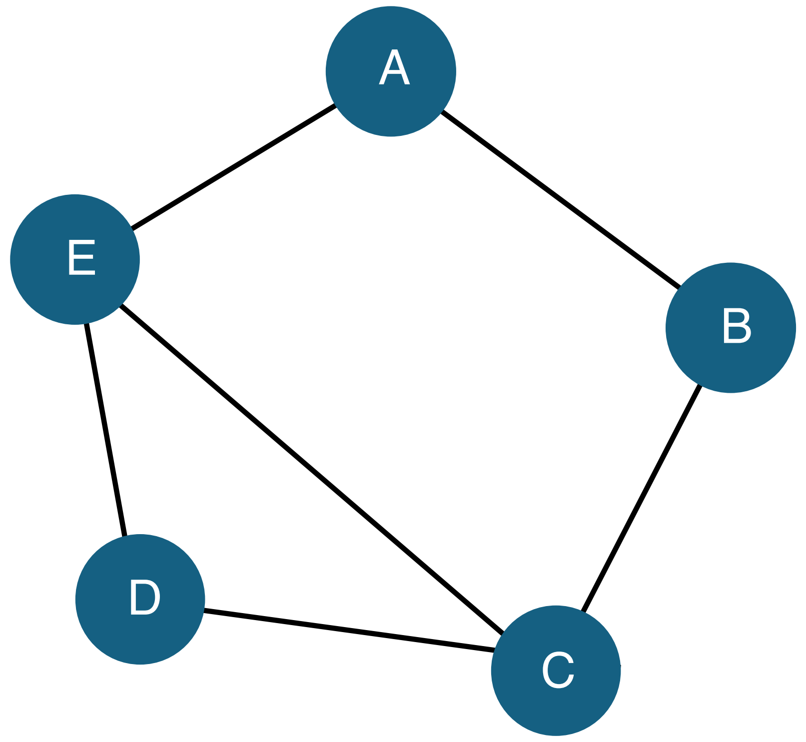

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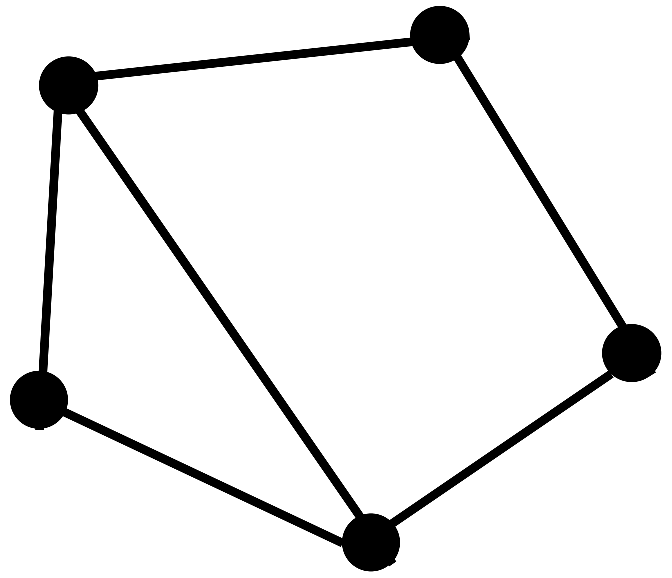

Communication steps ⟷ Two-body gates

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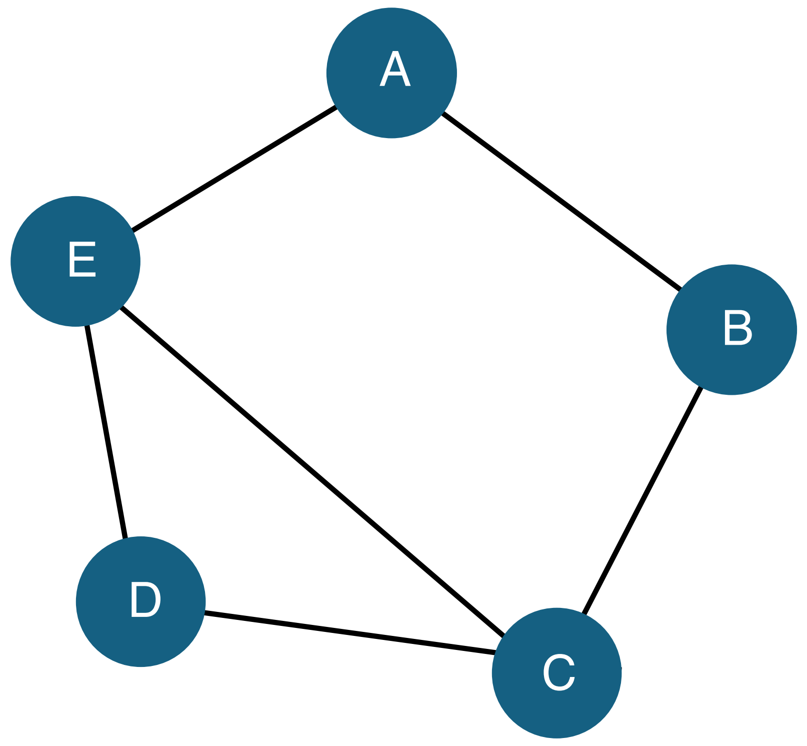
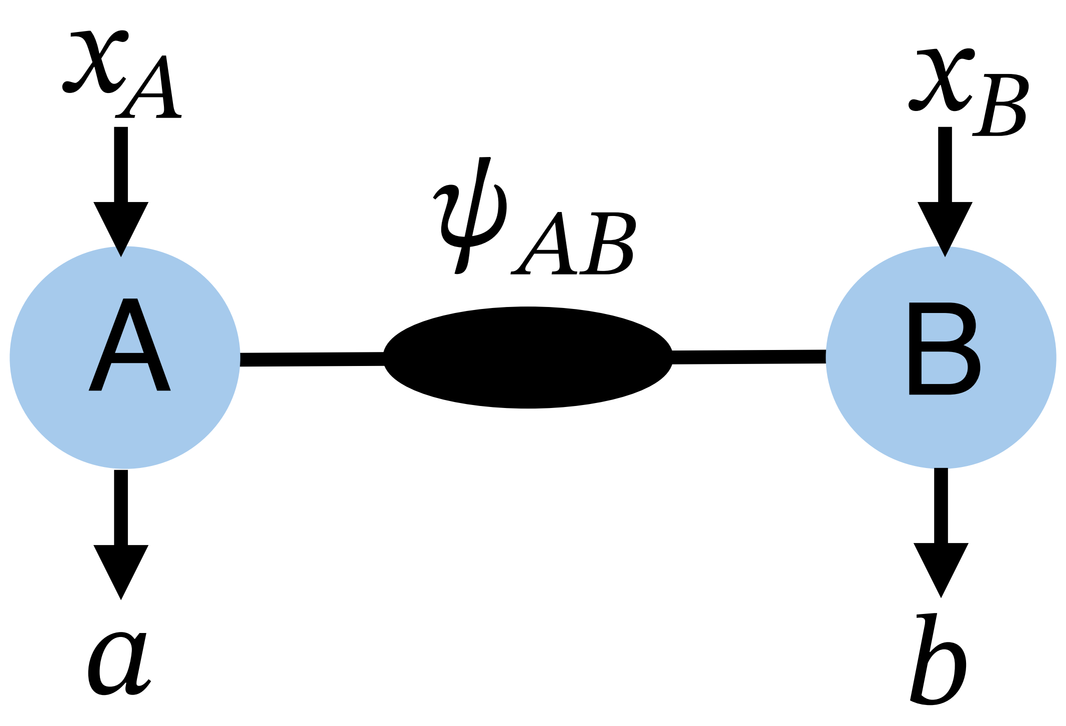


Communication steps ⟷ Two-body gates

💭 Potential fermionic advantage may emerge in distributed computing settings !

Review 2: CHSH Game: Qubits beat Classical bits



$$P(a, b \,|\, x_A, x_B)$$

Classical distribution cannot violate the Bell inequality.

$$a \oplus b = x_A \,.\, x_B$$

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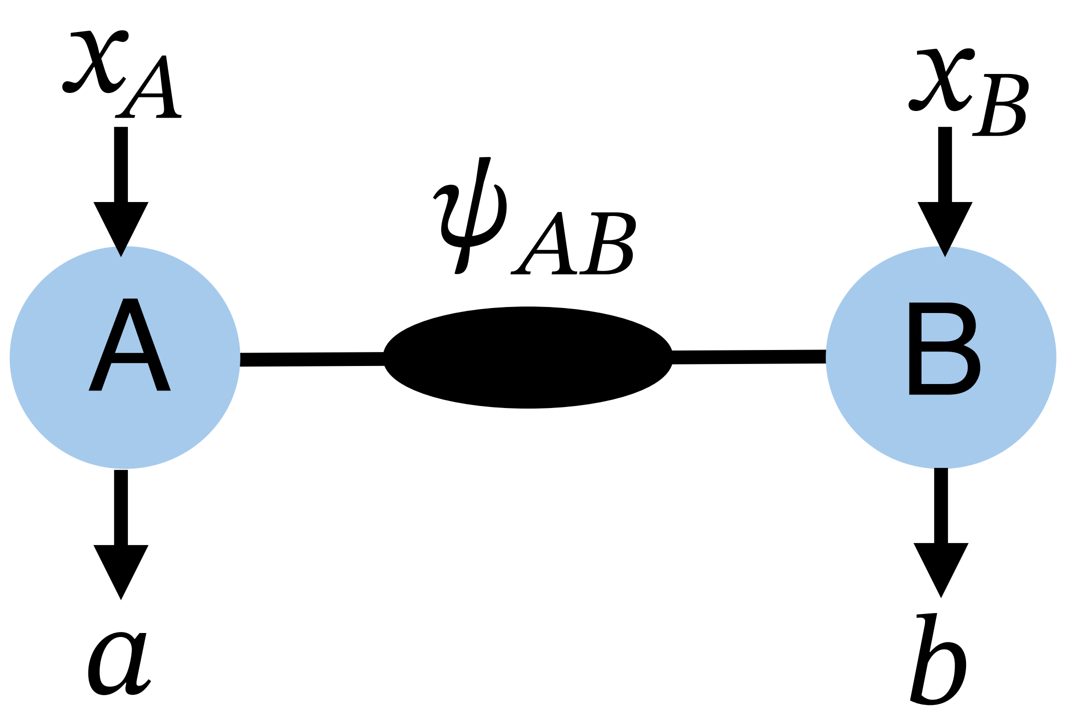

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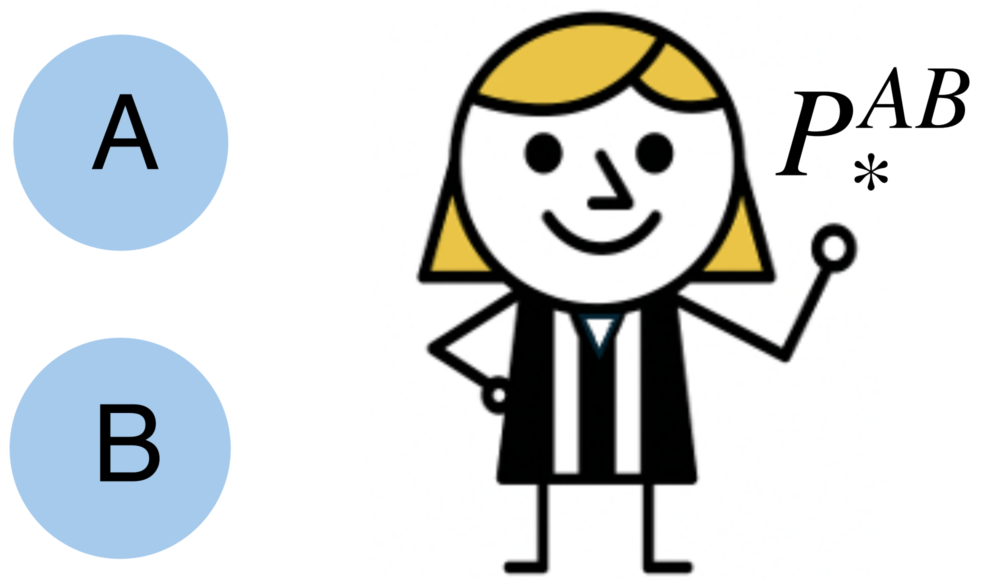

1. One round of communication
2. Receiving input bits
3. Outputting a bit

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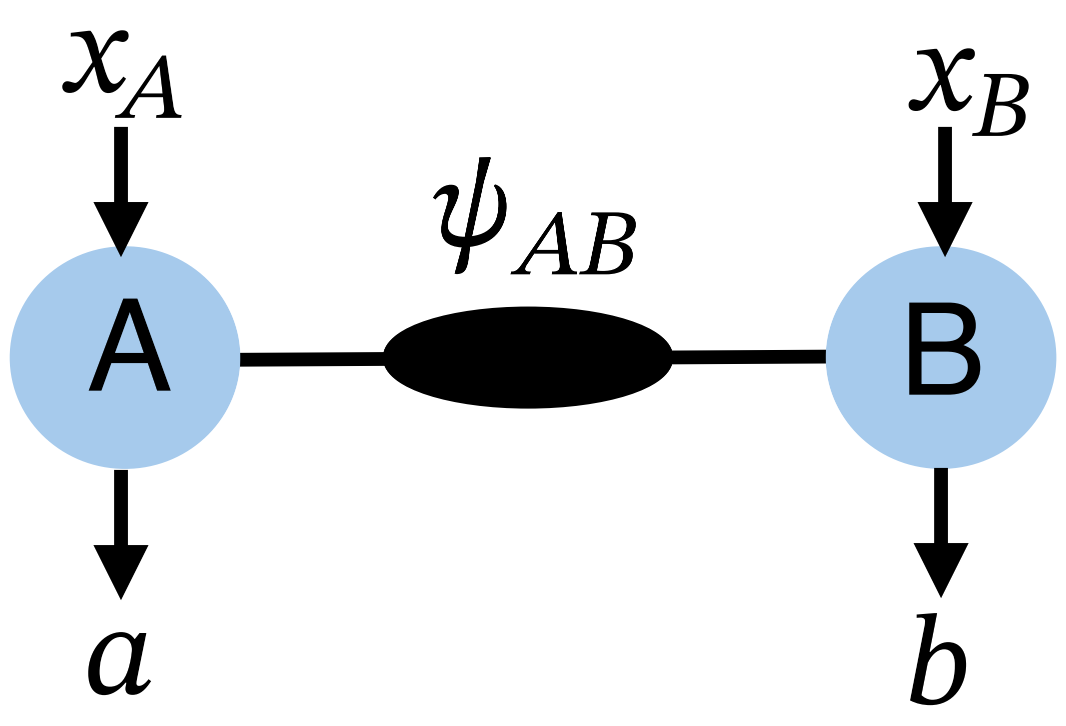

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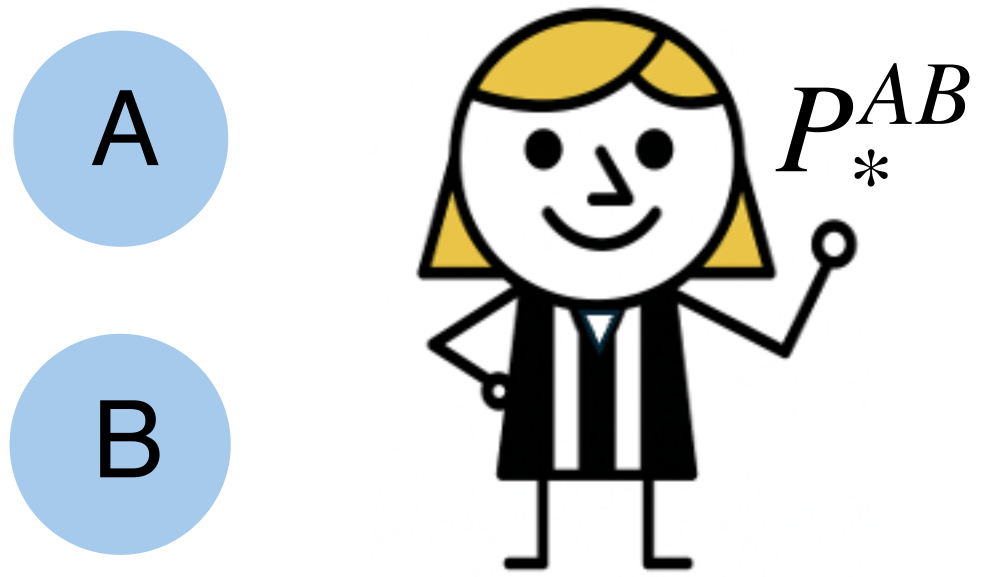

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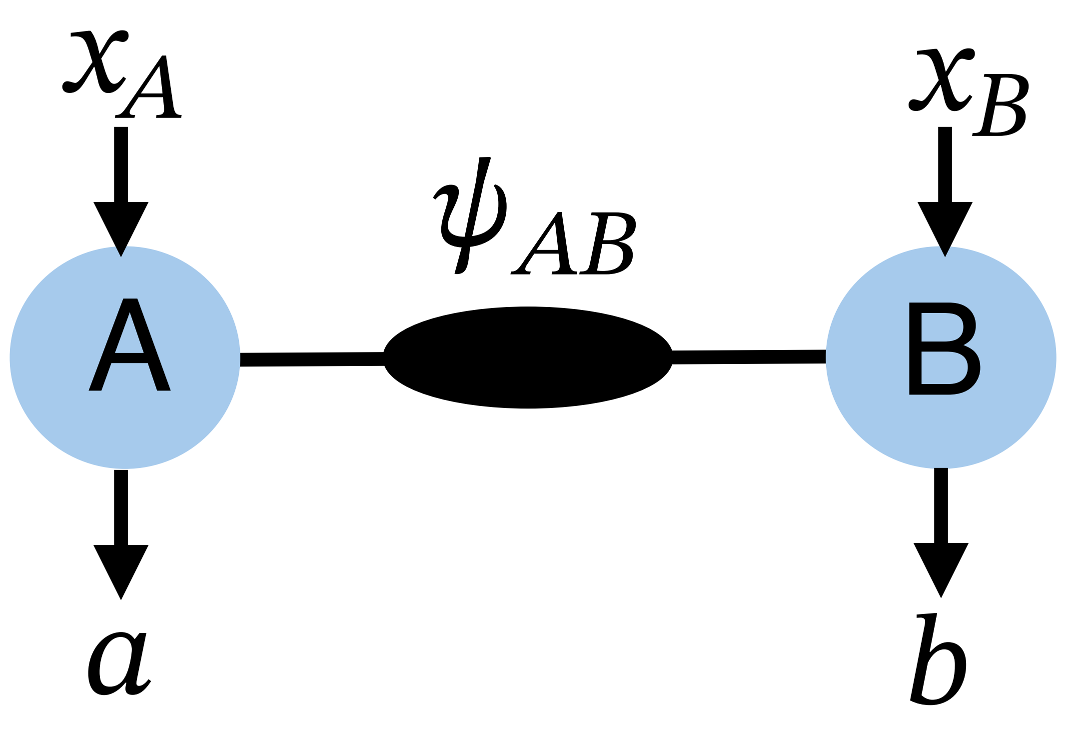

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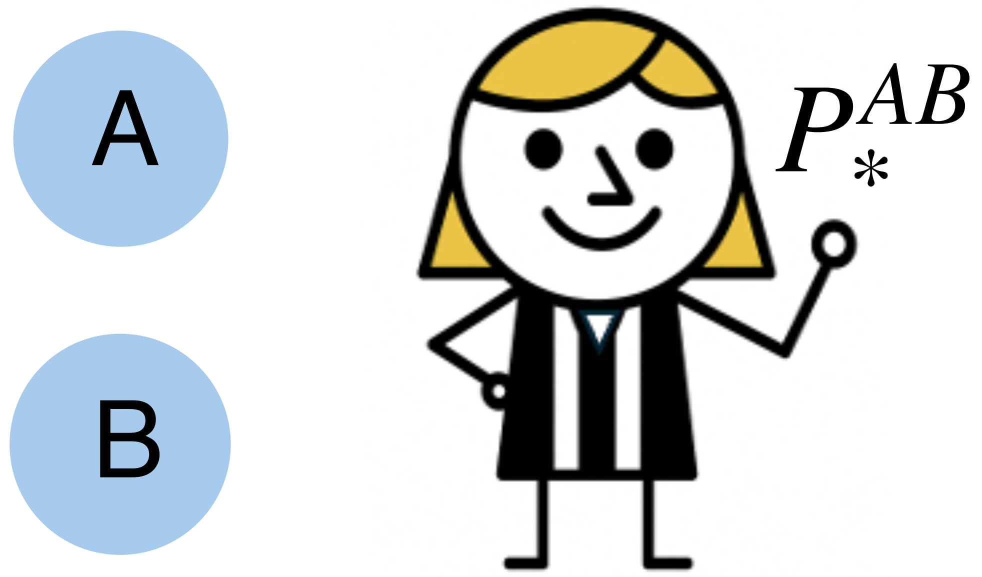

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What is the target distribution?

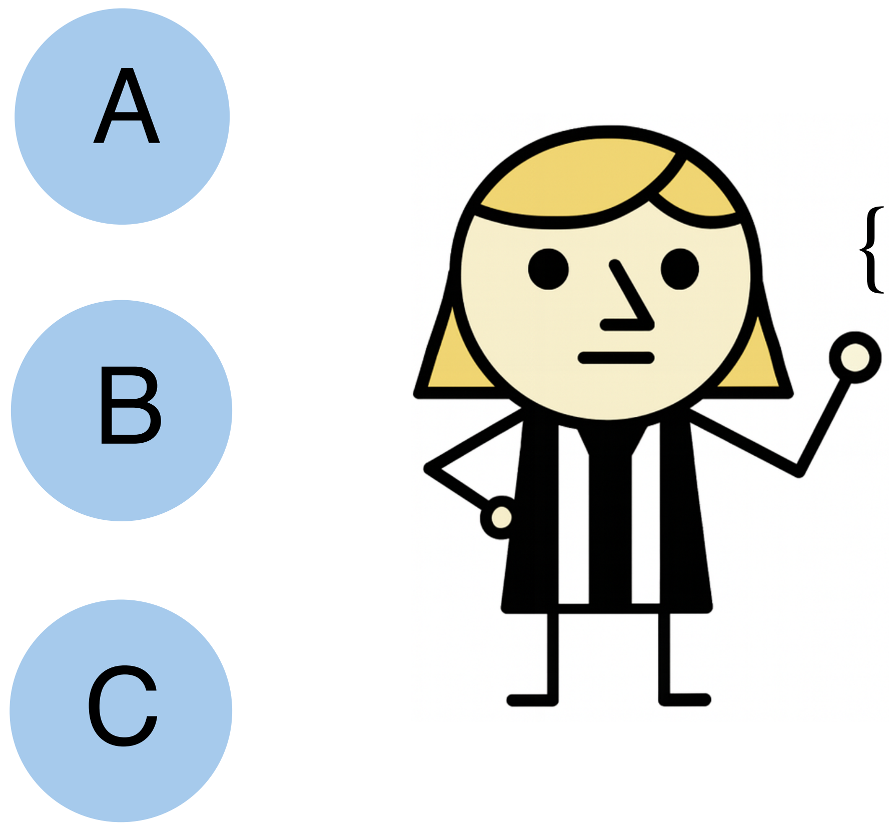
Qubit trusted strategy:

$$|\psi_{AB}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$M^{(A)}_{x_A=0} = X \qquad M^{(B)}_{x_B=0} = \frac{X+Z}{\sqrt{2}}$$

$$M^{(A)}_{x_A=1} = Z \qquad M^{(B)}_{x_B=1} = \frac{X-Z}{\sqrt{2}}$$

Our game: Febits beat Qubits



$\{P_*^{ABC}, P_*^{ACB}, P_*^{AB}, P_*^{AC}, P_*^{BC}\}$

1. The referee connects the parties
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4. Outputting a bit

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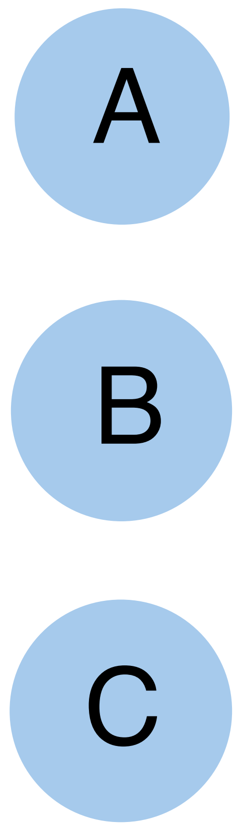

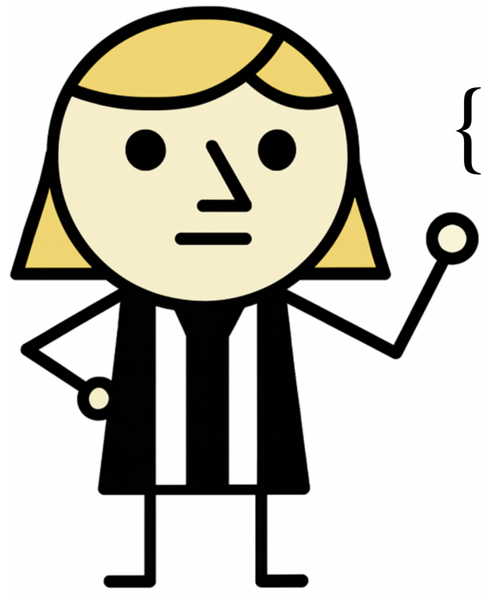

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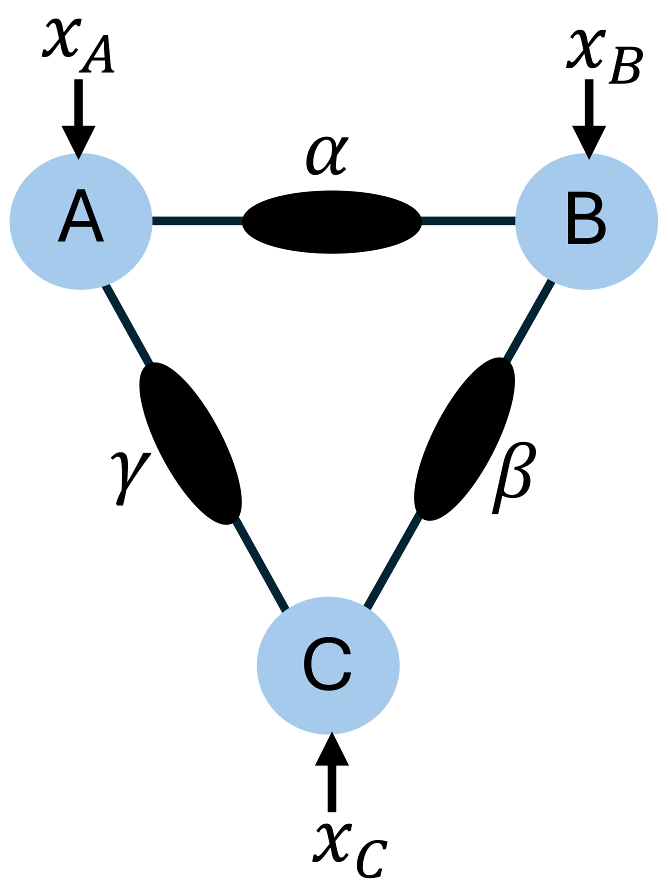

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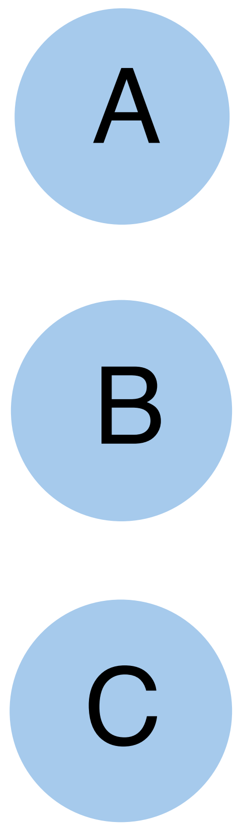

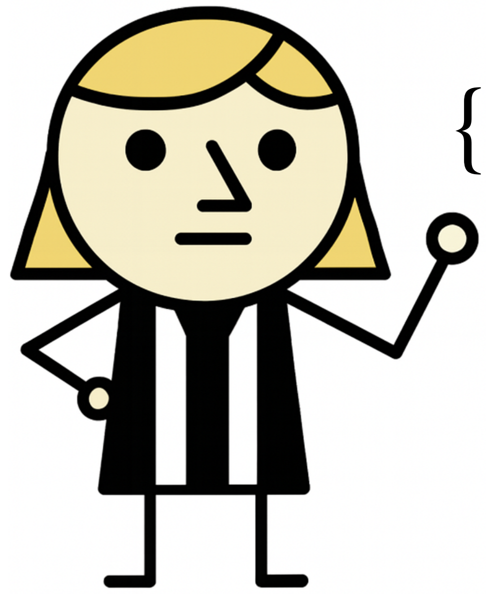

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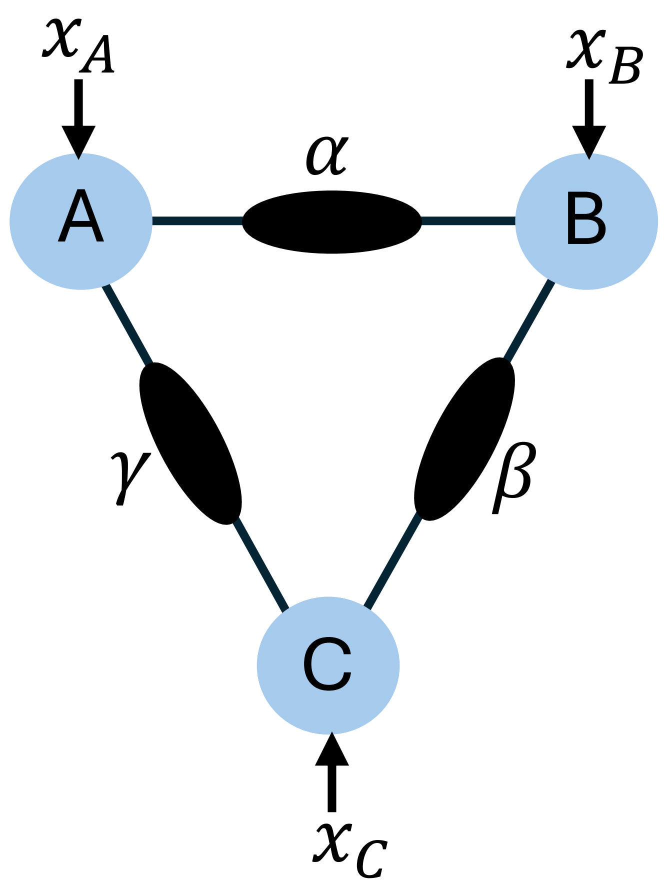

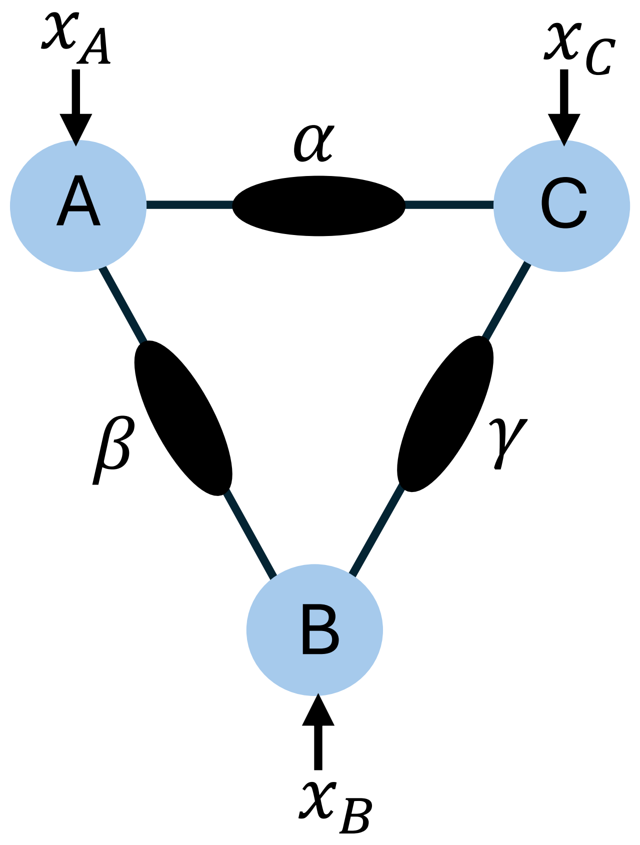

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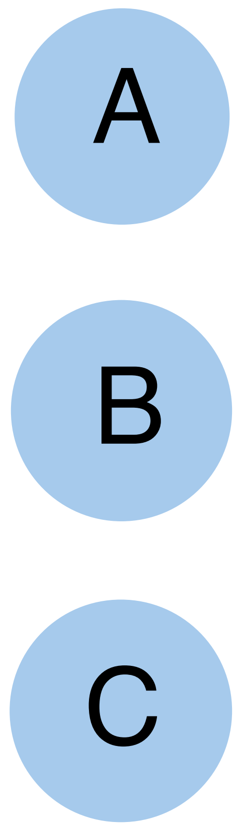

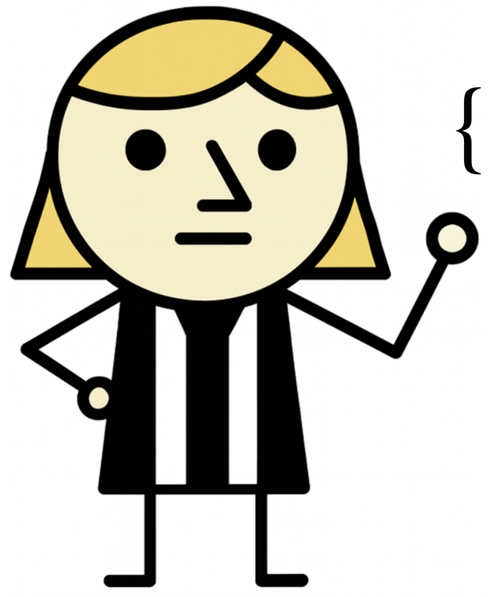

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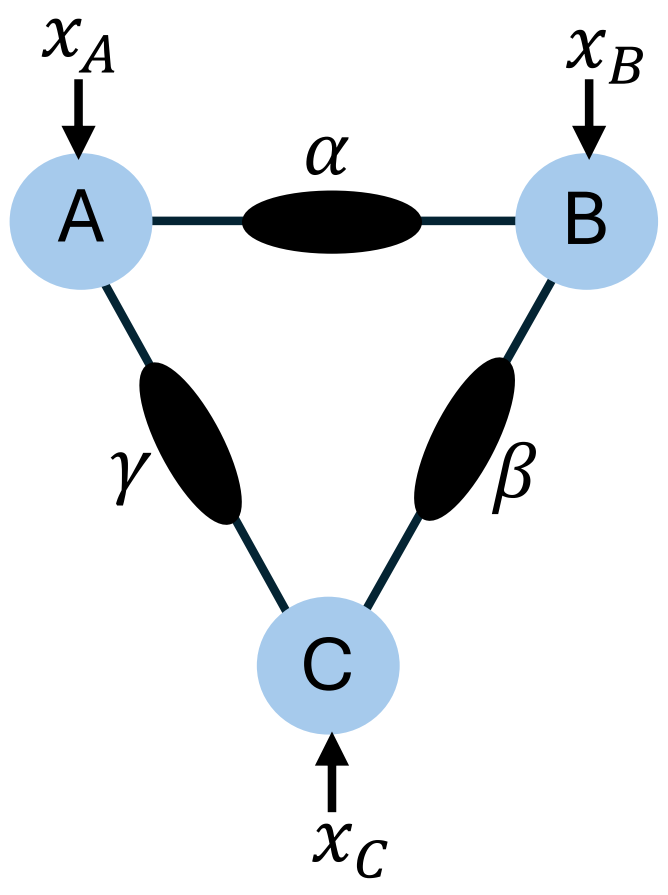

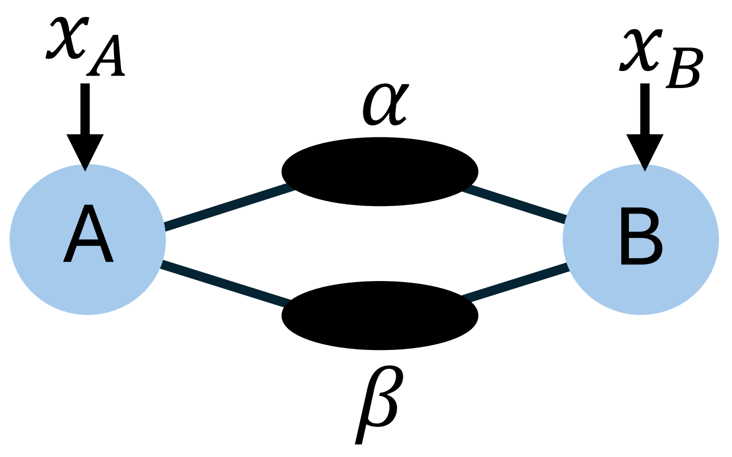

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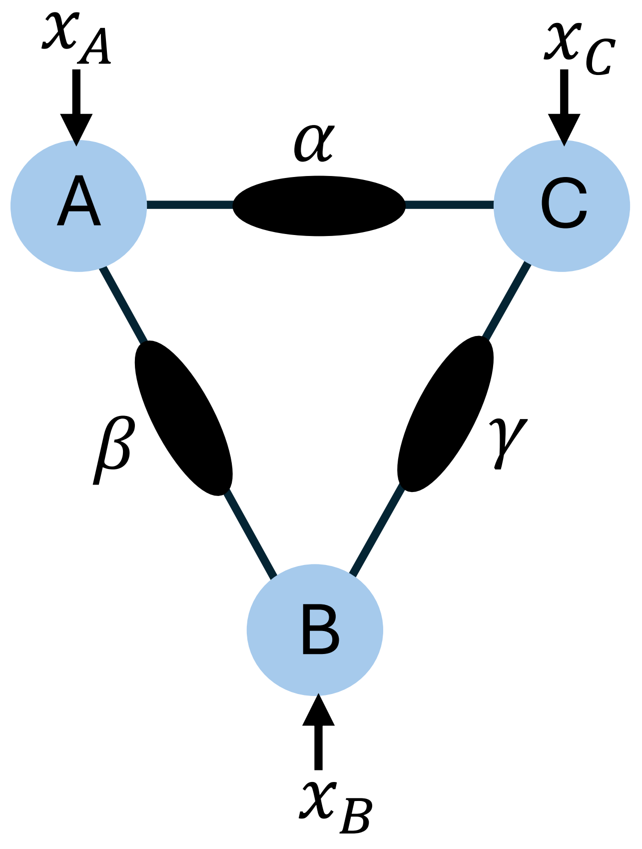

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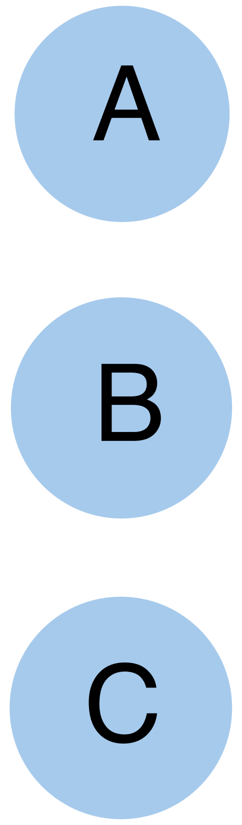

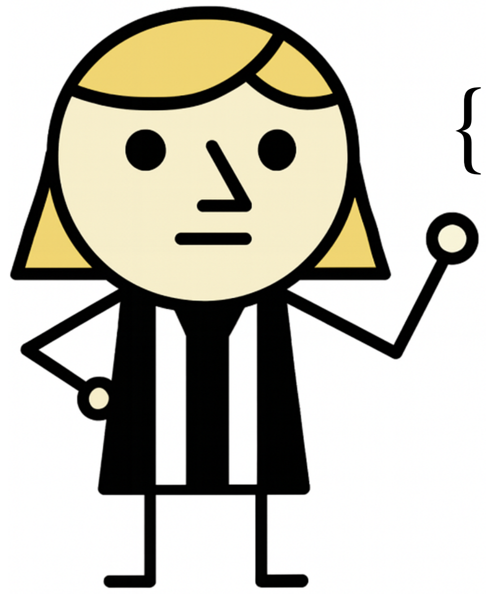

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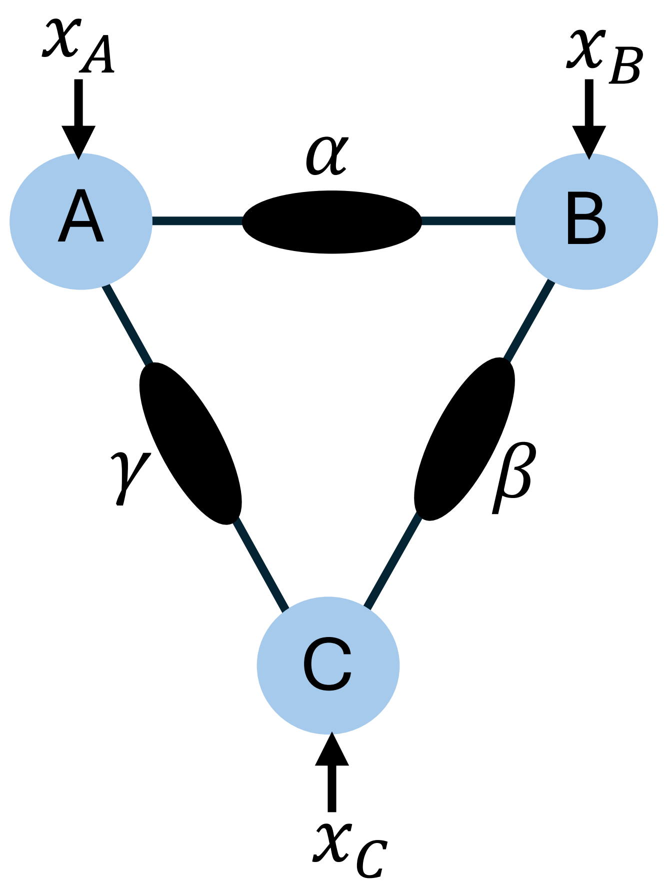
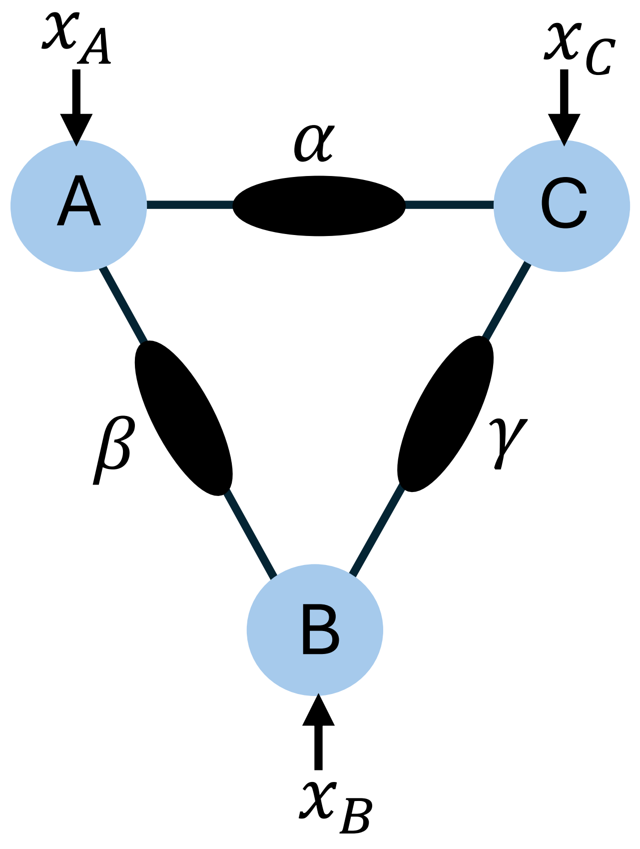
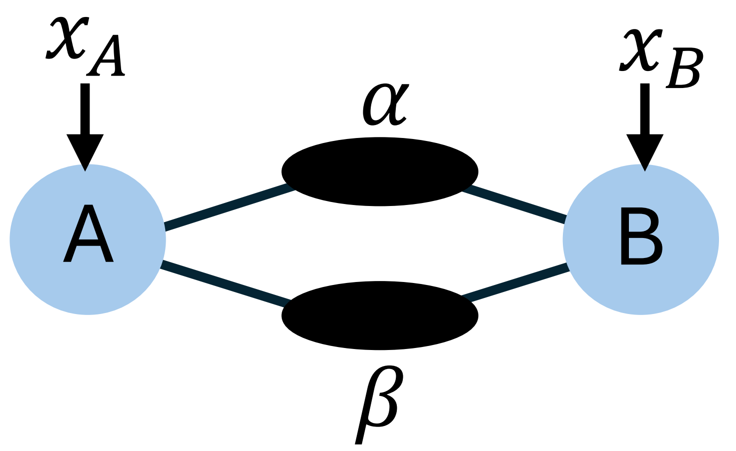
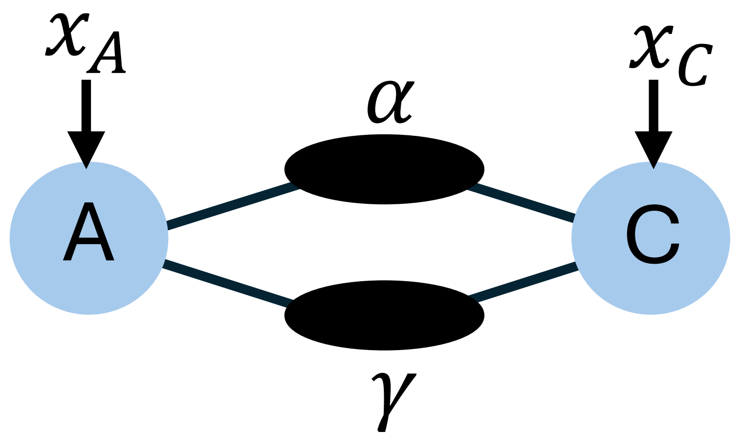
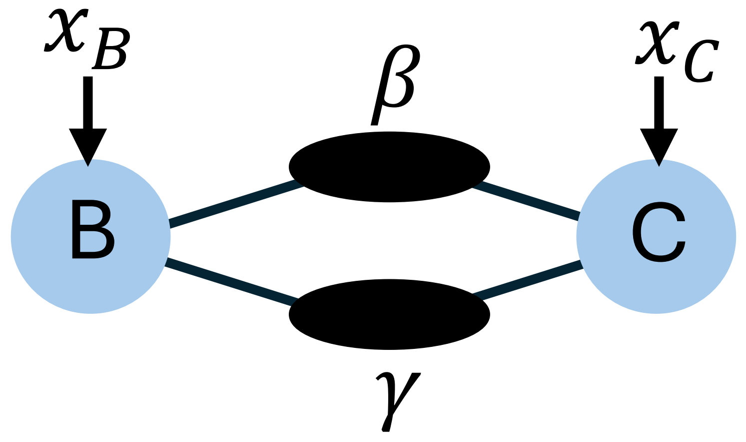


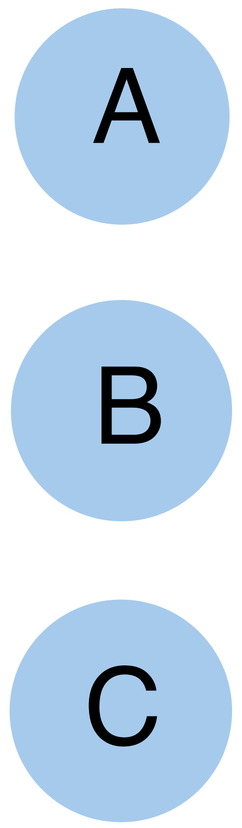


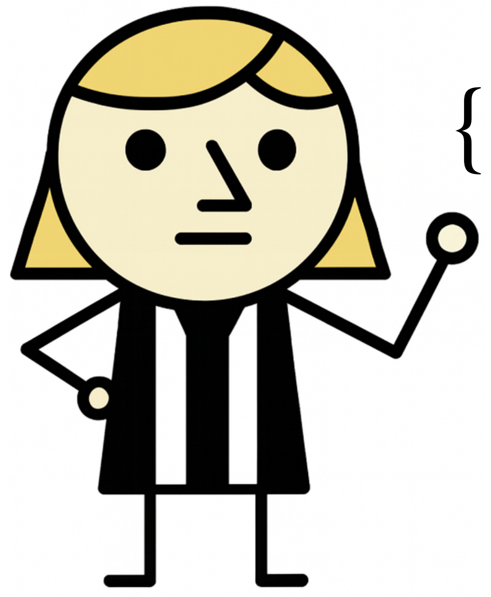






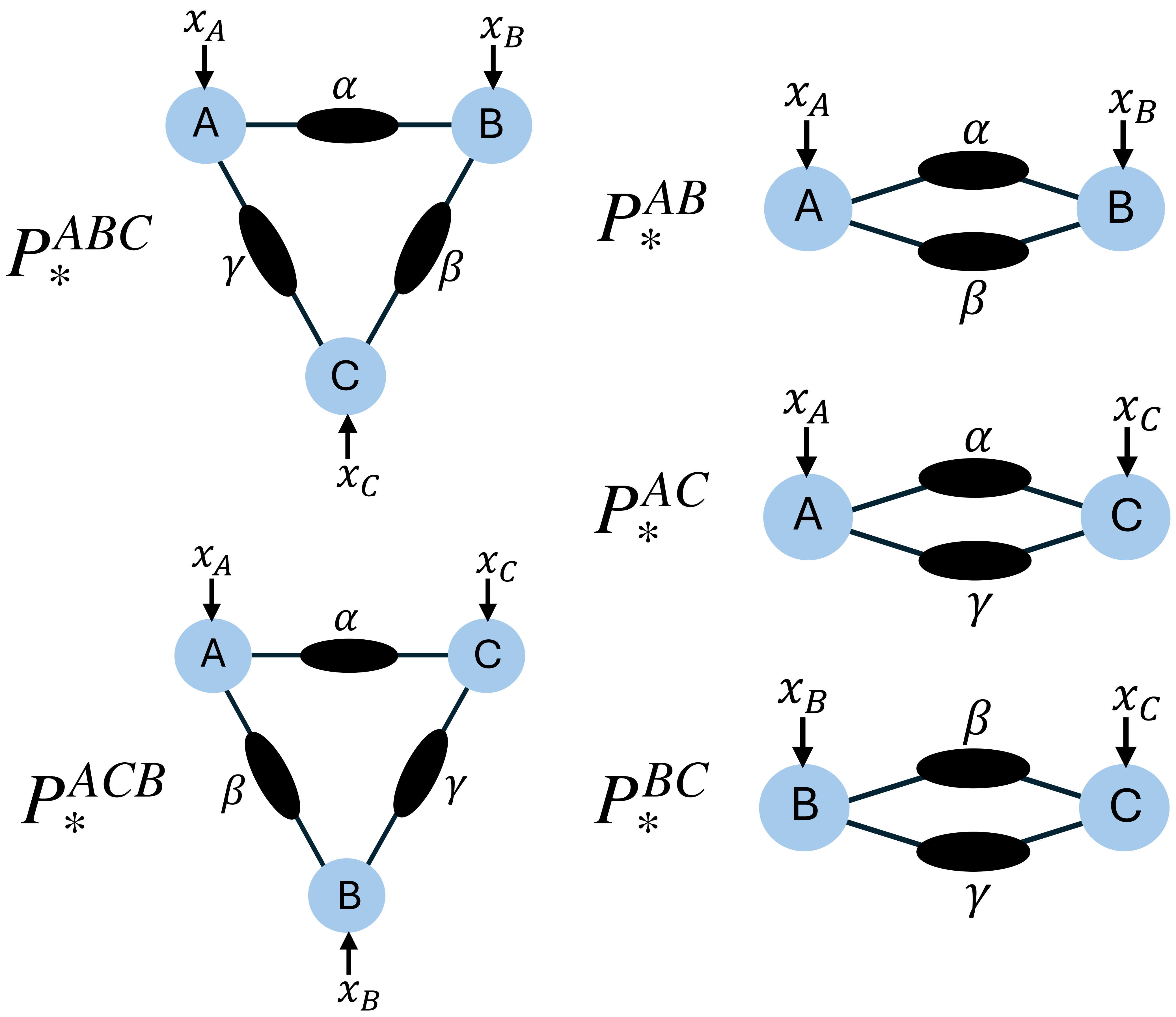
Our game: Febits beat Qubits





$\{P_*^{ABC}, P_*^{ACB}, P_*^{AB}, P_*^{AC}, P_*^{BC}\}$

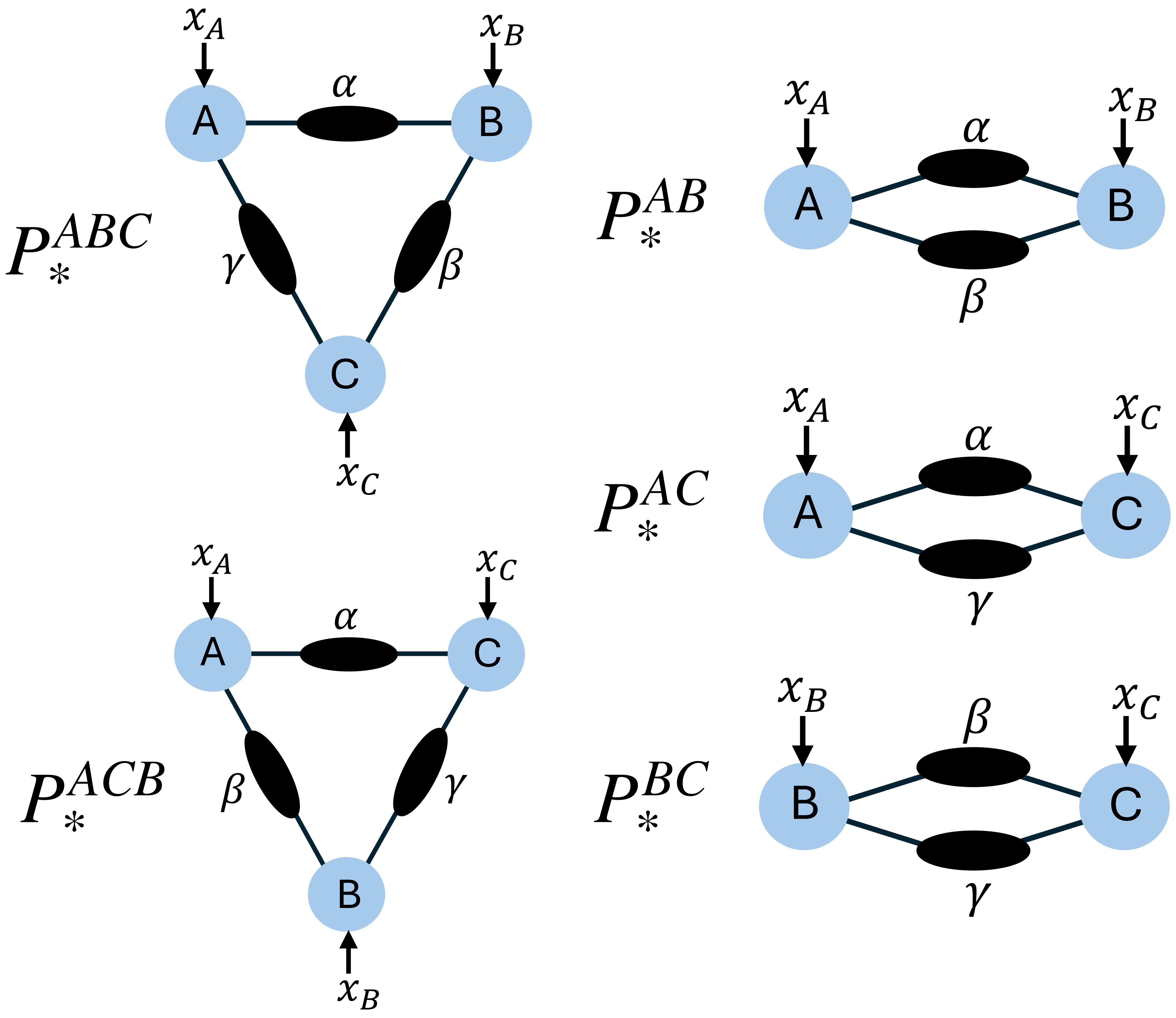
1. The referee connects the parties
2. One round of communication to left
3. Receiving input bits
4. Outputting a bit



Our game: Febits beat Qubits

What are the target distributions?

Fermionic trusted strategy:

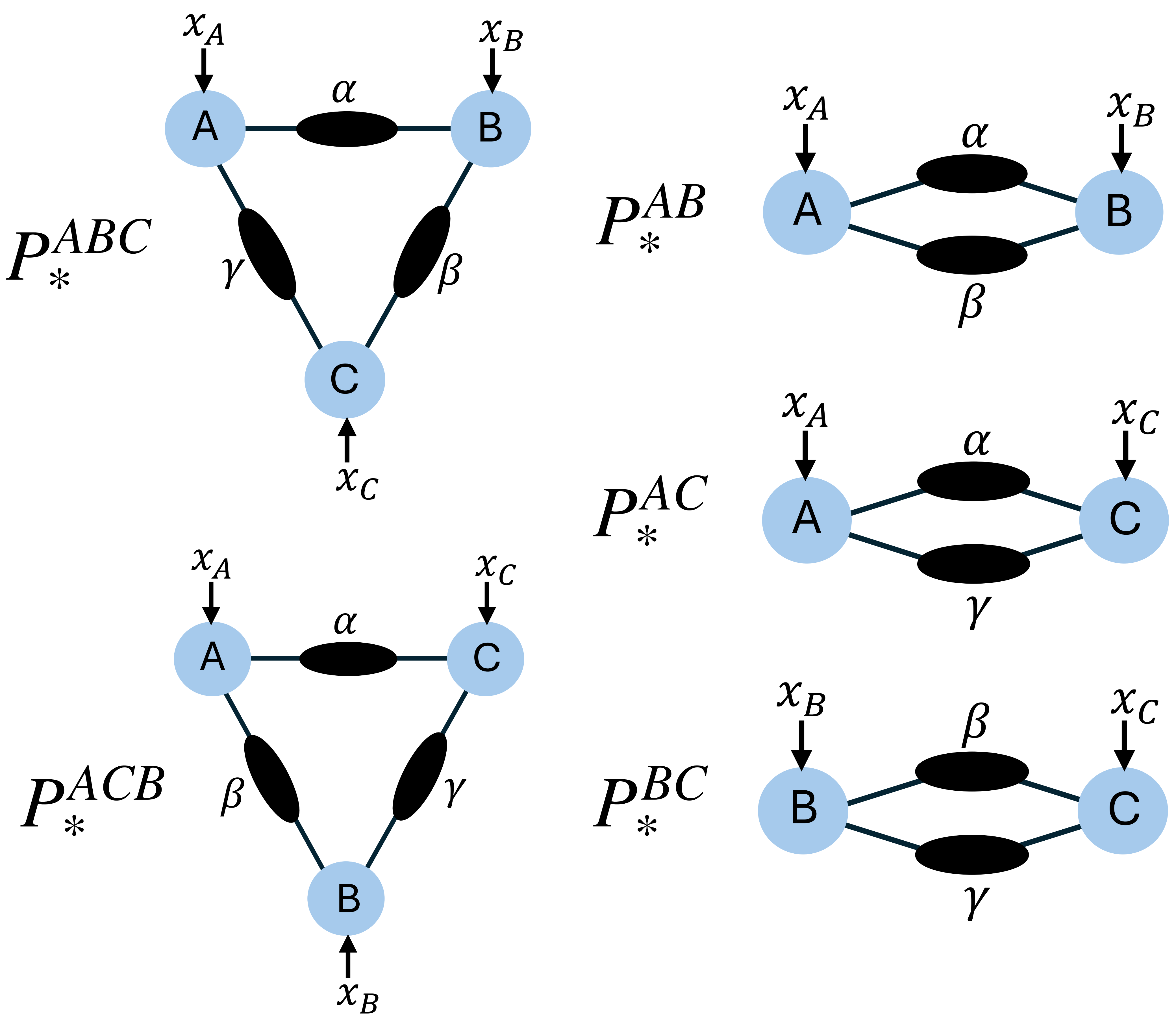


Our game: Febits beat Qubits

What are the target distributions?

Fermionic trusted strategy:

$$|\psi_\alpha\rangle = |\psi_\beta\rangle = |\psi_\gamma\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$



Our game: Febits beat Qubits

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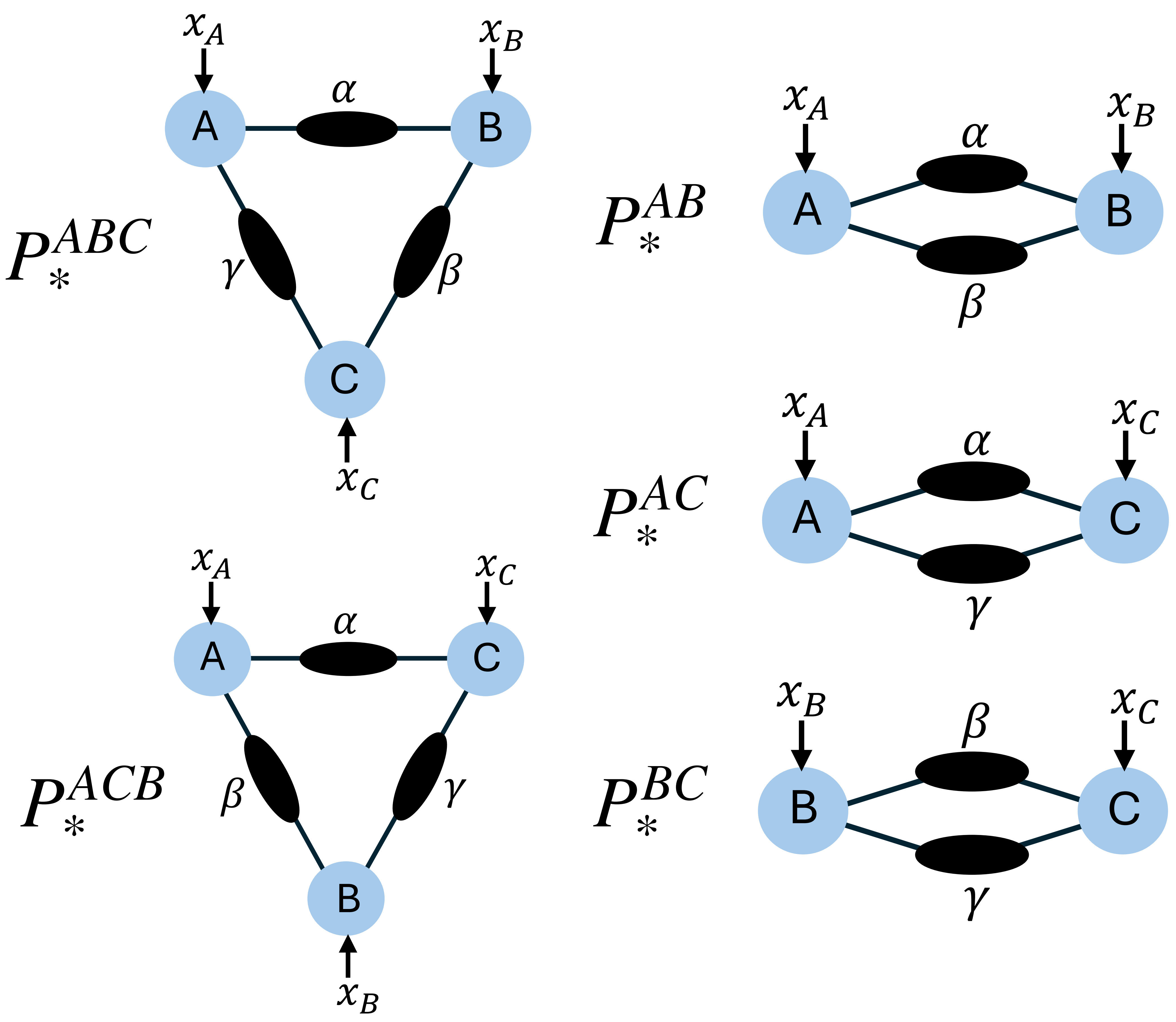
$$\begin{aligned} \Pi_i \;=\; \{\, \Pi_i^0 &= |00\rangle\langle 00|_{i_R i_L}, \\ \Pi_i^2 &= |11\rangle\langle 11|_{i_R i_L}, \\ \Pi_i^1 &= |01\rangle\langle 01|_{i_R i_L} + |10\rangle\langle 10|_{i_R i_L} \,\} \end{aligned}$$



Our game: Febits beat Qubits

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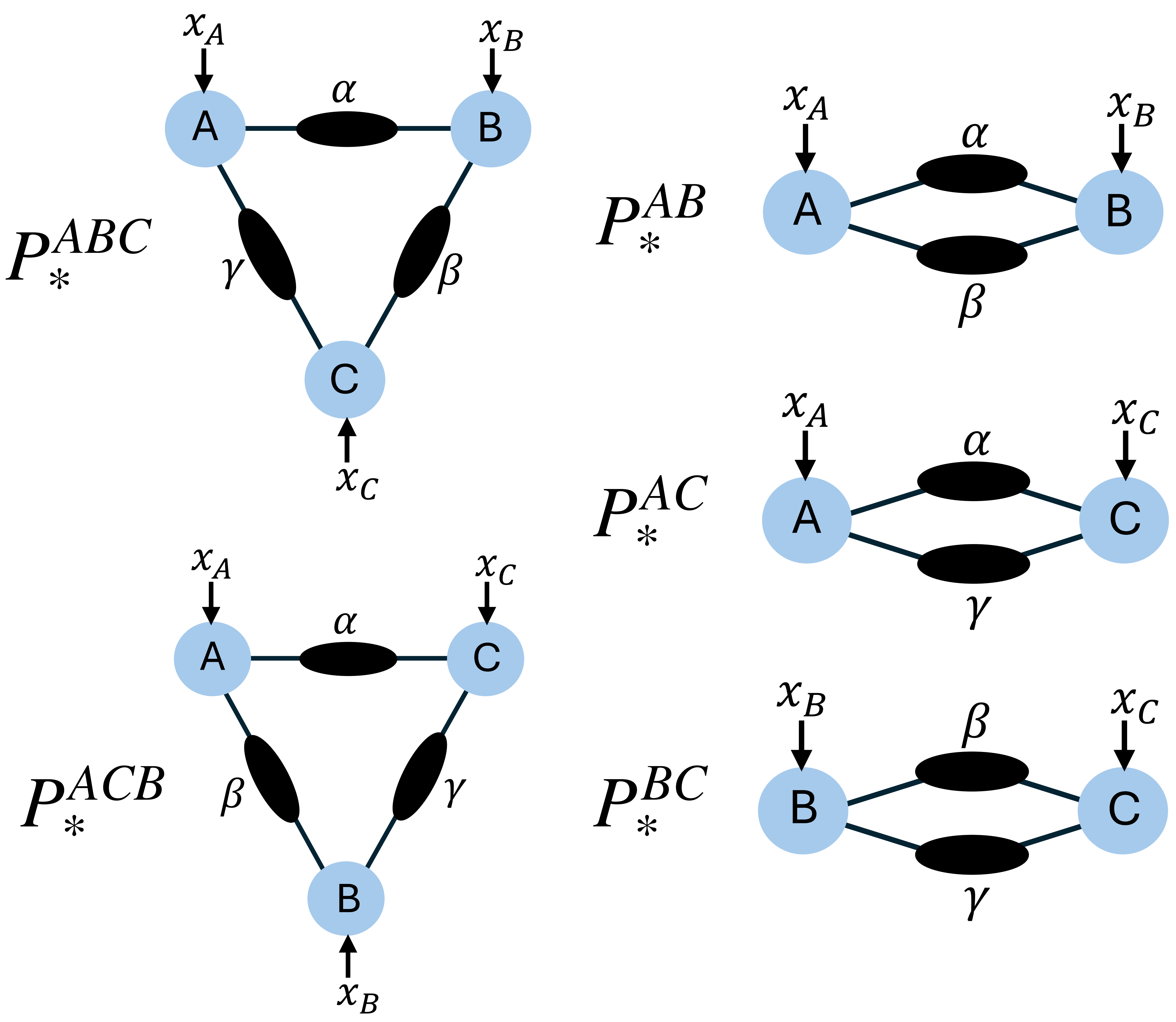
$$M_{x_i}^{(i)} = \left\{\, |m_{i,x_i}^+\rangle\langle m_{i,x_i}^+|_{i_R i_L},\, |m_{i,x_i}^-\rangle\langle m_{i,x_i}^-|_{i_R i_L} \,\right\}$$

$$|m_{i,x_i}^+\rangle = u_{i,x_i}^+\,|01\rangle_{i_R i_L} + v_{i,x_i}^+\,|10\rangle_{i_R i_L}$$

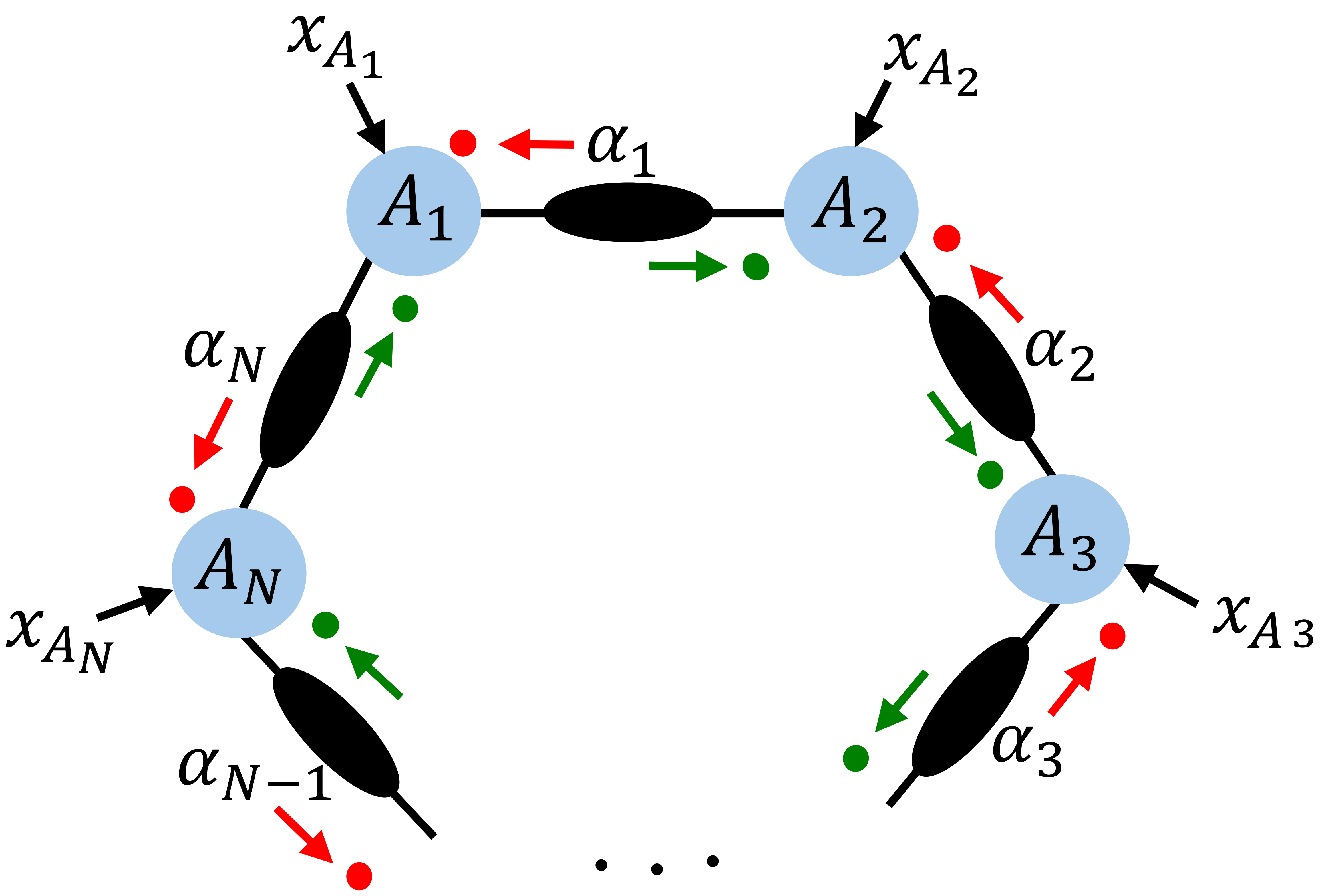
$$|m_{i,x_i}^-\rangle = u_{i,x_i}^-\,|01\rangle_{i_R i_L} + v_{i,x_i}^-\,|10\rangle_{i_R i_L}$$



Our game: Febits beat Qubits

Intuition of the proof

when everyone obtains 1 particle:



$$|\psi\rangle_b = \frac{|01,\ldots,01\rangle + |10,\ldots,10\rangle}{\sqrt{2}}$$

$$|\psi\rangle_f = \frac{|01,\ldots,01\rangle + (-1)^{N-1}|10,\ldots,10\rangle}{\sqrt{2}}$$

Our game: Febits beat Qubits

Intuition of the proof

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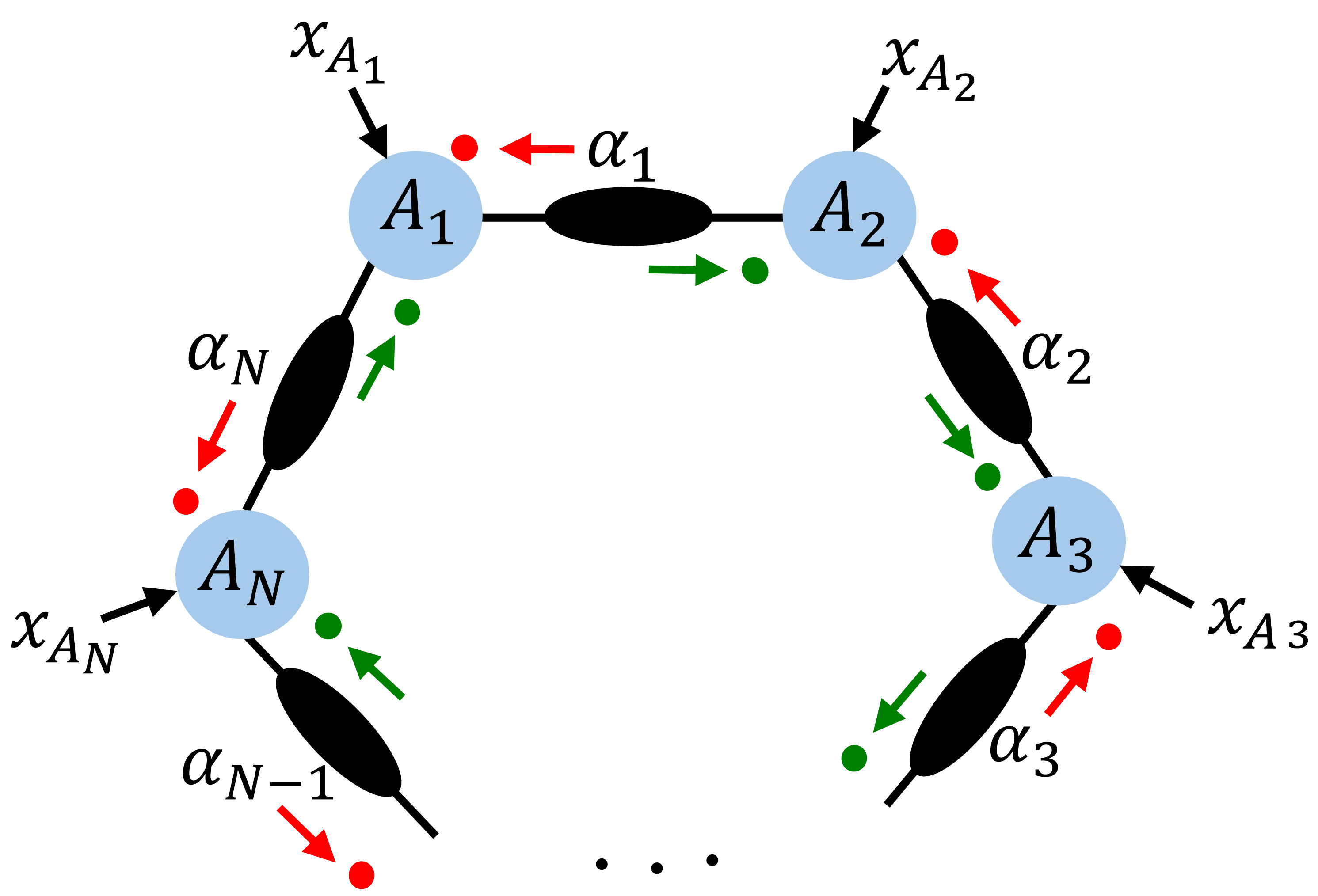

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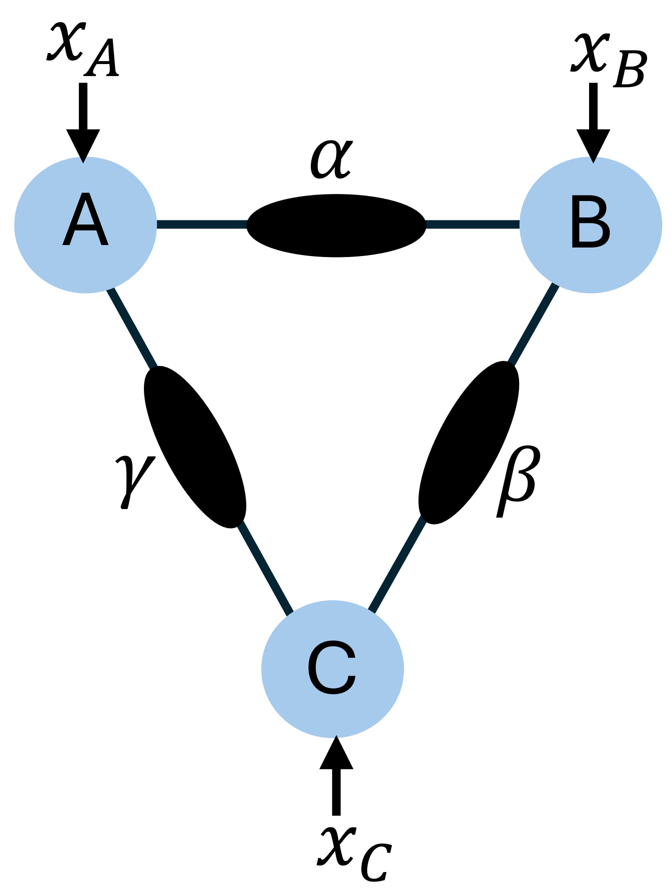
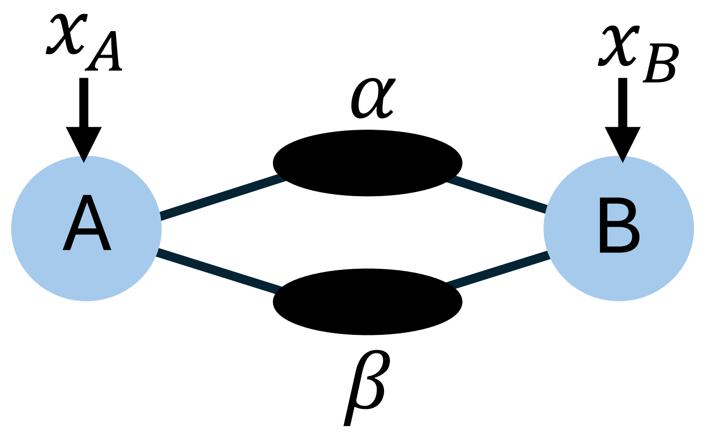


$$|\psi\rangle_b = |GHZ_3^+\rangle$$
$$|\psi\rangle_f = |GHZ_3^+\rangle$$



$$|\psi\rangle_b = |GHZ_2^+\rangle$$
$$|\psi\rangle_f = |GHZ_2^-\rangle$$

Future Directions

- Comparing the fermionic game with the bosonic/distinguishable particle (Qubit) games with more than one step of communication.
- Finding a practical computational task.

Future Directions

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Thank you!

Superfast Bravy Kitaev

$$Z_j^{(F)} \longmapsto \prod_{k:(j,k)\in E} Z_{jk}^{(Q)}$$

$$X_j^{(F)} X_k^{(F)} \longmapsto \epsilon_{jk} X_{jk} \prod_{p:(j,p)<_j(j,k)} Z_{jp} \prod_{q:(k,q)<_k(k,j)} Z_{kq}$$

Other mappings

Fermionic to Qubit mappings

$$|n_1 n_2 \ldots n_m\rangle_F := (f_1^\dagger)^{n_1}(f_2^\dagger)^{n_2}\cdots(f_m^\dagger)^{n_m}|\Omega\rangle$$

2. Parity:

$$|n_1 n_2 \ldots n_m\rangle_F \longmapsto |q_1 q_2 \ldots q_m\rangle_Q, \qquad q_j = n_1 \oplus n_2 \oplus \cdots \oplus n_j$$

$$f_j^\dagger = \frac{1}{2}\big(Z_{j-1}X_j - iY_j\big)\prod_{k=j+1}^{m} X_k, \quad f_j = \frac{1}{2}\big(Z_{j-1}X_j + iY_j\big)\prod_{k=j+1}^{m} X_k$$

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Fermionic to Qubit mappings

$$|n_1 n_2 \ldots n_m\rangle_F := (f_1^\dagger)^{n_1}(f_2^\dagger)^{n_2} \cdots (f_m^\dagger)^{n_m}|\Omega\rangle$$

3. Bravy Kitaev:

$$|n_1 n_2 \ldots n_m\rangle_F \longmapsto |q_1 q_2 \ldots q_m\rangle_Q$$

- Some qubits store the occupation numbers, some store partial parities.

Parity super-selection rule: no signaling

$|+,0\rangle_{12}$ $f_2 + f_2^\dagger$

$$(f_2 + f_2^\dagger)|+,0\rangle_{12} = \frac{1}{\sqrt{2}}(f_2 + f_2^\dagger)(|0,0\rangle_{12} + |1,0\rangle_{12}) =$$

$$\frac{1}{\sqrt{2}}(|0,1\rangle_{12} - |1,1\rangle_{12}) = |-,1\rangle_{12}$$

Quantum Mechanics

1. First Quantization Picture

Distinguishable Particles: $|\psi\rangle_1 \in \mathscr{H}_1, \quad |\phi\rangle_2 \in \mathscr{H}_2 \quad \rightarrow \quad |\Psi\rangle_{12} = |\psi\rangle_1 \otimes |\phi\rangle_2$

Indistinguishable Particles:

1. Bosons $$|\Psi\rangle_{\mathrm{b}} = \frac{1}{\sqrt{2}}\Big(|\psi\rangle \otimes |\phi\rangle + |\phi\rangle \otimes |\psi\rangle \Big)$$

2. Fermions $$|\Psi\rangle_{\mathrm{f}} = \frac{1}{\sqrt{2}}\Big(|\psi\rangle \otimes |\phi\rangle - |\phi\rangle \otimes |\psi\rangle \Big)$$

2. Second Quantization Picture

Distinguishable particles/Bosons: $[a_i, a_j^\dagger] = \delta_{ij}\, I, \quad [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$

Fermions: $\{f_i, f_j^\dagger\} = \delta_{ij} I, \quad \{f_i, f_j\} = \{f_i^\dagger, f_j^\dagger\} = 0$