# A friendly introduction to Distributed Algorithms & Locality … through two examples

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# Distributed algorithms

- You write a program for ***one computer***
  - you can use also commands like “send this message to this communication port” etc.
- Your adversary constructs ***a network of n computers***, all running your program
- Switch everything on, see what happens

# Distributed algorithms

- Useful abstraction: identical computers, working in a synchronous manner
- Running time = ***number of communication rounds***
- Key challenge: what to do in the middle of a very large network?

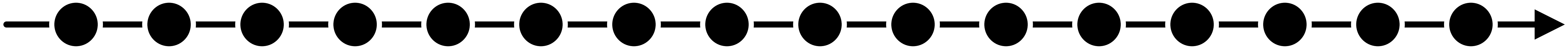
# 1

*Theme:*

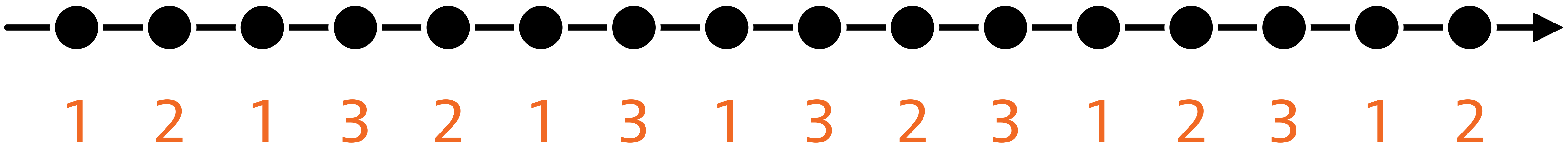
**Symmetry breaking**

*Example:*

**3-coloring cycles**

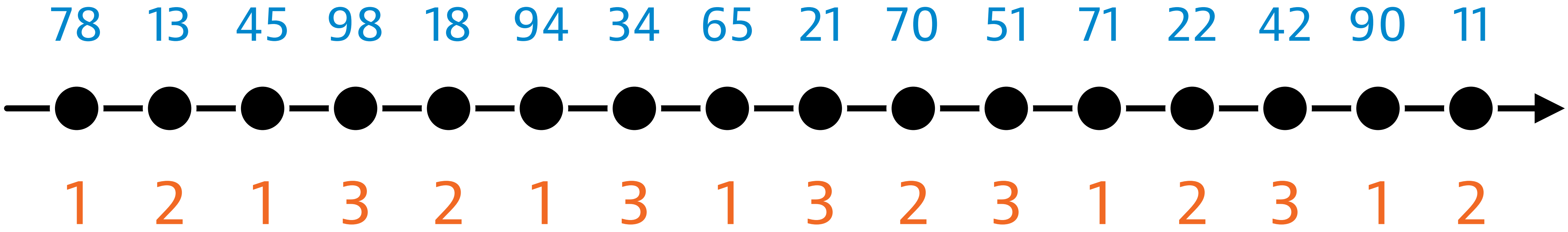


**Computer network: directed cycle**
*All nodes have a well-defined "successor" and "predecessor"*




**Coloring:**

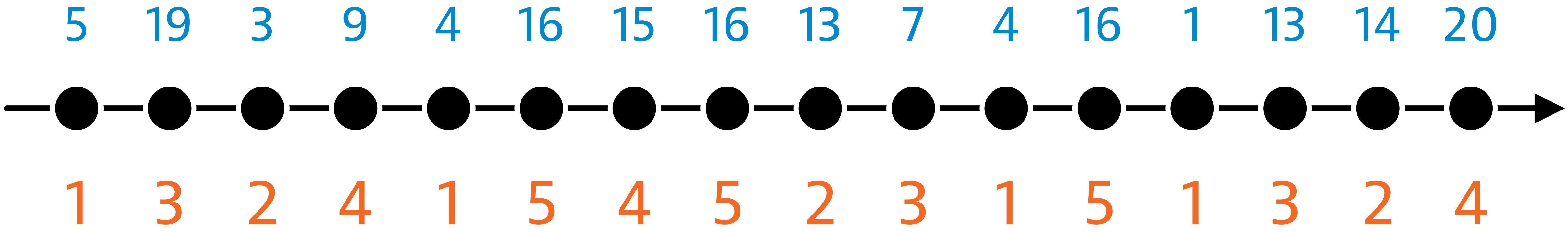
*how to break symmetry?*




**Color reduction:**

*unique identifiers or random strings*

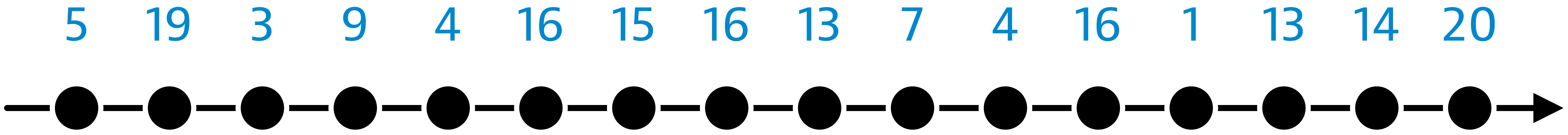
→ poly($n$) colors → 3 colors




**One color reduction step:**

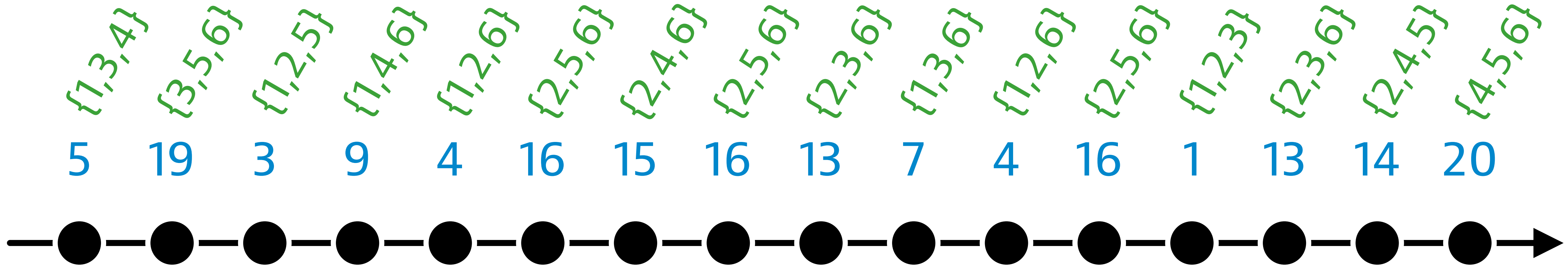
20 colors → 6 colors

???




**One color reduction step:**

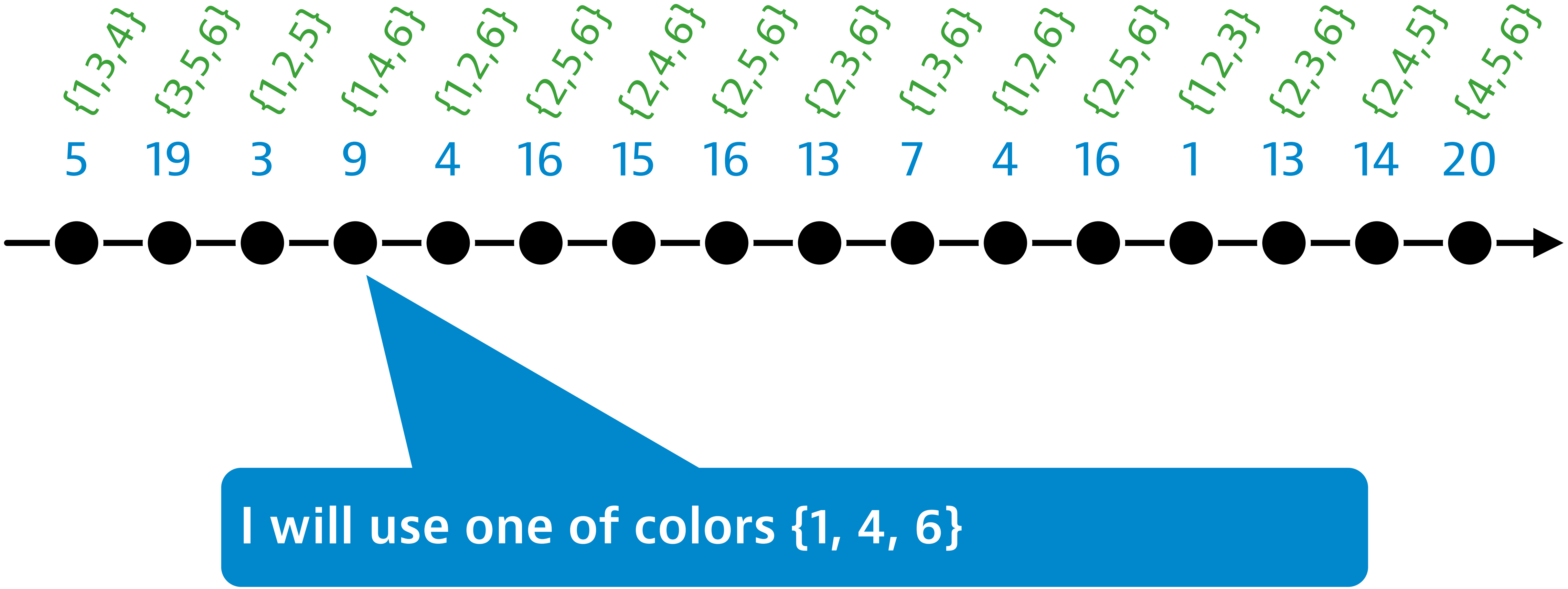
20 colors → …

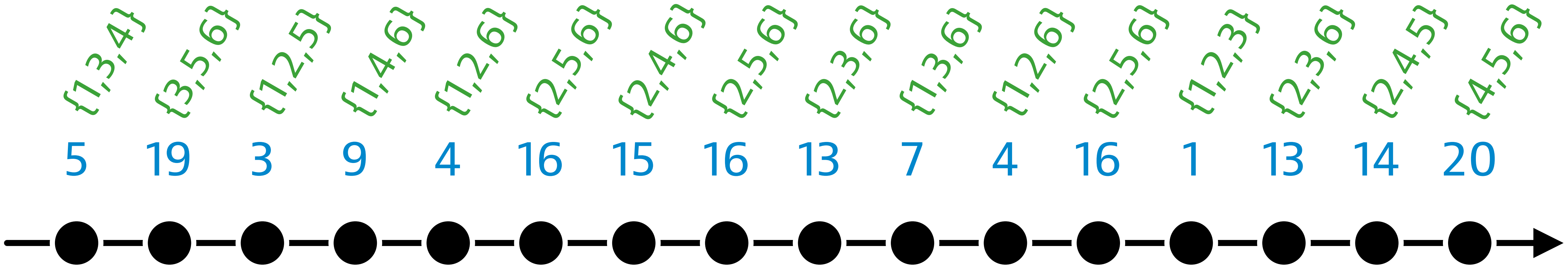



**One color reduction step:**

20 colors → 3-element subsets of {1, 2, ..., 6}

→ …

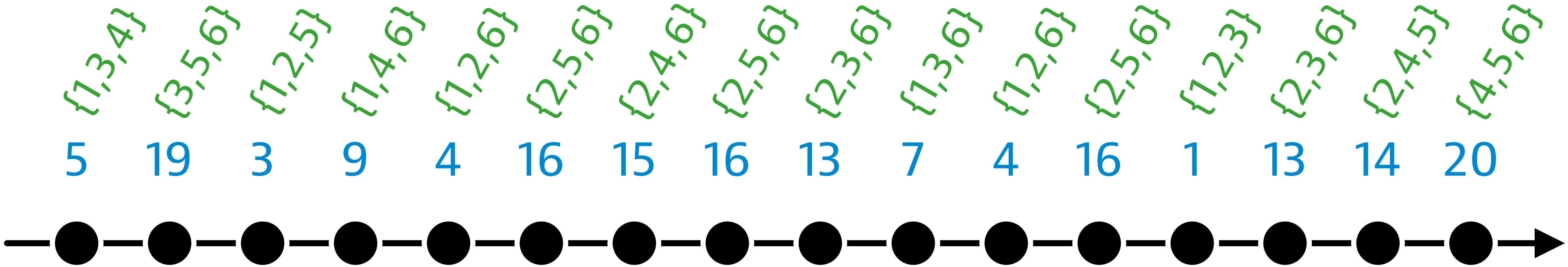

{1,3,4} {3,5,6} {1,2,5} {1,4,6} {1,2,6} {2,5,6} {2,4,6} {2,5,6} {2,3,6} {1,3,6} {1,2,6} {2,5,6} {1,2,3} {2,3,6} {2,4,5} {4,5,6}
5 19 3 9 4 16 15 16 13 7 4 16 1 13 14 20
I will use one of colors {1, 4, 6}




**One color reduction step:**

20 colors → 3-element subsets of {1, 2, ..., 6}

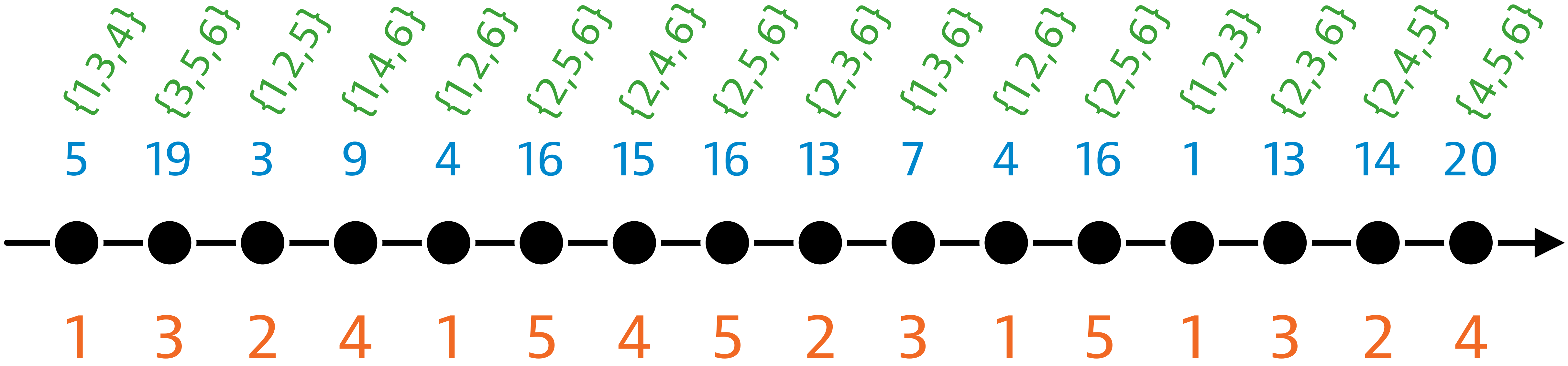
→ one communication round → ...




I promised to use one of colors {1, 4, 6}

My successor will use one of colors {1, 2, 6}

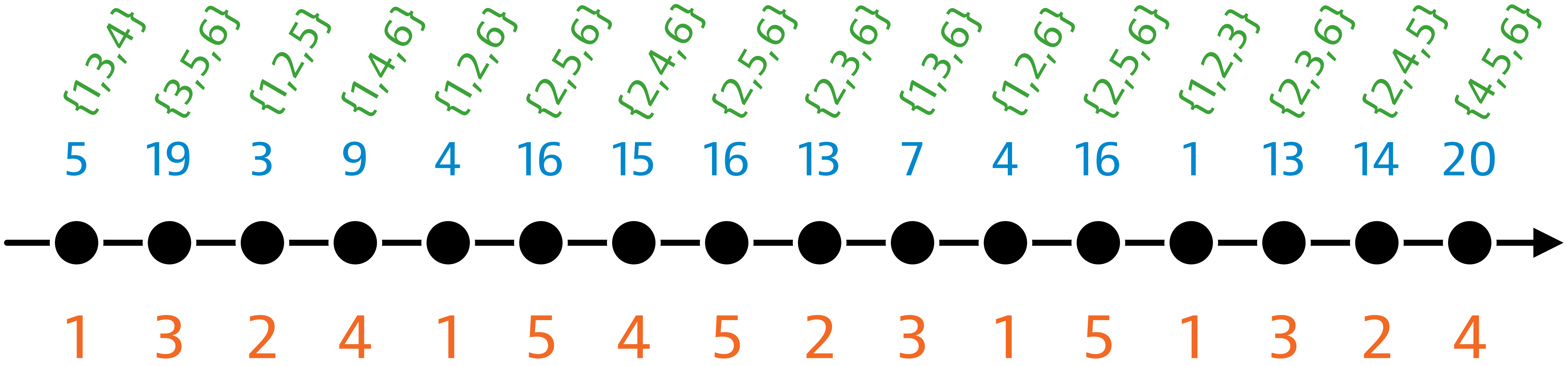
I can therefore safely output 4




**One color reduction step:**

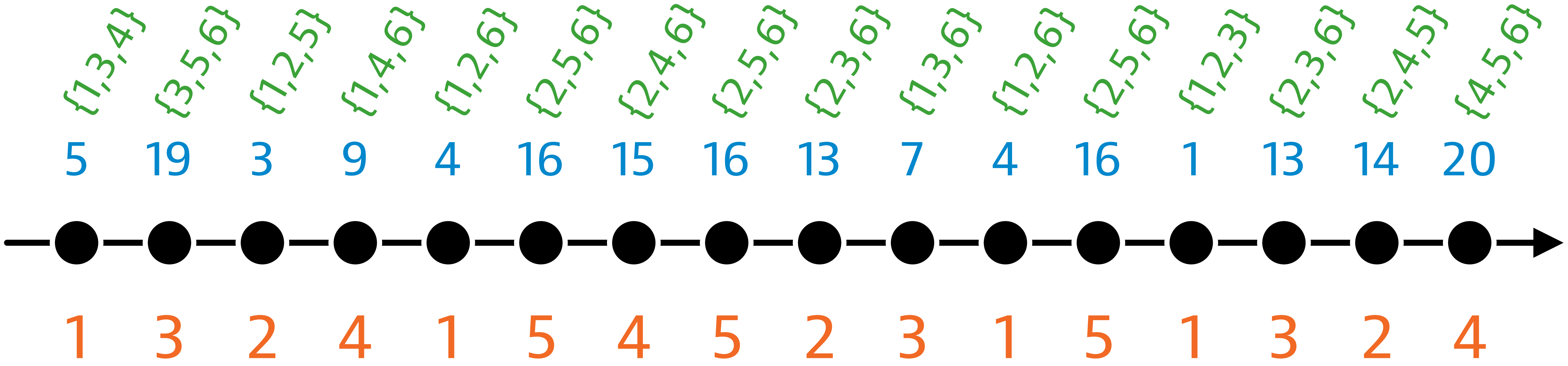
20 colors → 3-element subsets of {1, 2, ..., 6}

→ one communication round → 6 colors




**One color reduction step:**

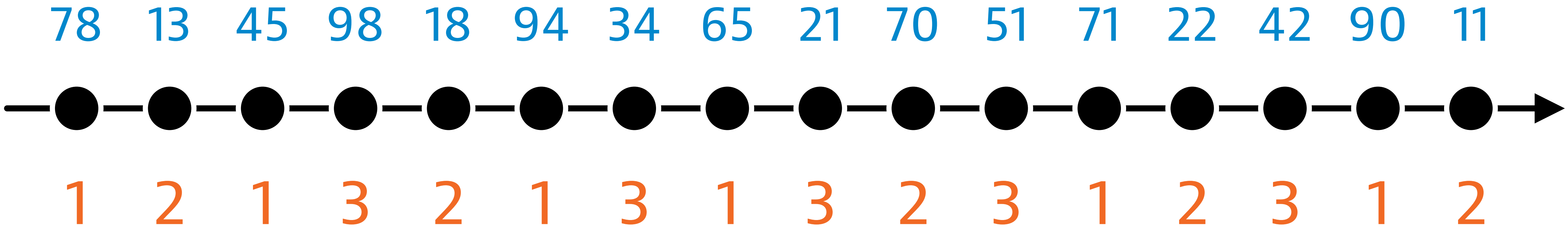
(2*k* choose *k*) colors → 2*k* colors




**One color reduction step:**

($2k$ choose $k$) colors → $2k$ colors

$c$ colors → $O(\log c)$ colors




**Color reduction:**

*unique identifiers or random strings*

→ poly($n$) colors

→ $O(\log^* n)$ color reduction steps

→ 3 colors

- **Algorithm:** $O(\log^* n)$ rounds
  - ***color reduction:*** $c$ colors in $T$ rounds
    → $O(\log c)$ colors in $T + 1$ rounds
  - poly($n$) colors in 0 rounds
    → 3 colors in $O(\log^* n)$ rounds
- **Lower bound:** this is optimal!
  - ***round elimination:*** c colors in $T$ rounds
    → $2^c$ colors in $T - 1$ rounds
  - 3 colors in $o(\log^* n)$ rounds
    → ≪ poly($n$) colors in 0 rounds → impossible

# Locality

Each node knows its final output
after ***T* communication rounds**

$$\Updownarrow$$

Each node can compute its final output
if it knows its ***T*-radius neighborhood**

# What about quantum?

- **Classical:** $\approx \frac{1}{2} \log^* n$
- **Non-signaling:** $\approx 1$
- **Quantum:** major open question that cannot be resolved by using arguments related to causality / finitely dependent / non-signaling!

# Some concrete numbers

- You can reduce the number of colors from **$10^{70}$** to **10** in one round
- **Implication:** one-round **classical** algorithm for 10-coloring with an astronomically small failure probability!
- **Challenge:** numerical experiments related to **quantum** advantage??

2

*Theme:*

**Local coordination**

*Example:*

**Maximal matching**

# Maximal matching

- **Setting:**
  - 2-colored graph — "black" and "white" nodes
  - maximum degree $\Delta$
- **Goal:**
  - maximal matching: if you are unmatched, all your neighbors must be matched

# Proposal algorithm

- **Black nodes:**
  - send a proposal to 1$^{st}$ neighbor
  - if rejected, send a proposal to 2$^{nd}$ neighbor …
- **White nodes:**
  - wait for proposals
  - accept the first proposal
  - reject all other proposals

- **Algorithm:** $O(\Delta)$ rounds
  - ***proposal algorithm:*** after $\Delta$ failed attempts you run out of neighbors to propose to and can safely output "unmatched"
- **Lower bound:** this is optimal!
  - ***round elimination:*** maximal matching in $T$ rounds → a sloppy matching in $T - 1$ rounds
  - maximal matching in $o(\Delta)$ rounds
    → something nontrivial in 0 rounds
    → contradiction

# What about quantum?

- **Known:** the only quantum lower bound that we have is very weak & indirect:
  - quantum doesn't help with ***linear programs***
  - lower bounds for fractional maximum matching approximation apply [KMW 2004 etc.]
  - roughly **log Δ** rounds required
- **Open:** o(Δ)-round quantum algorithms?

# Summary

**3-coloring cycles: symmetry breaking**

- trivial sequential algorithm
- $O(\log^* n)$-round distributed algorithm
- classically tight, by round elimination

**Maximal matching: local coordination**

- trivial sequential algorithm
- $O(\Delta)$-round distributed algorithm
- classically tight, by round elimination