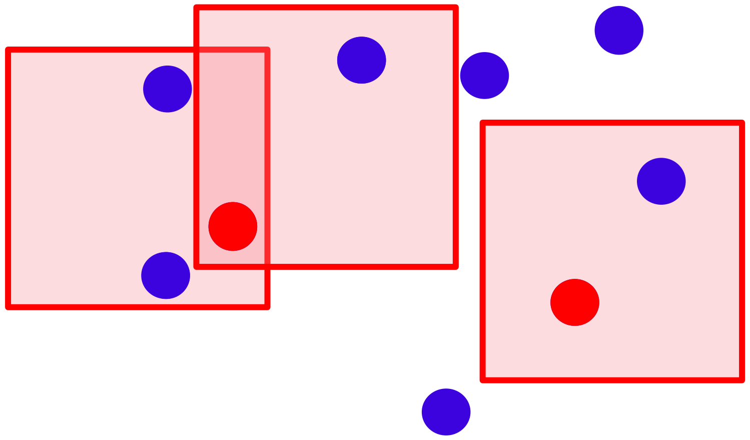
ONLINE HITTING SET FOR VARIOUS GEOMETRIC OBJECTS



HALT Frontier Friday
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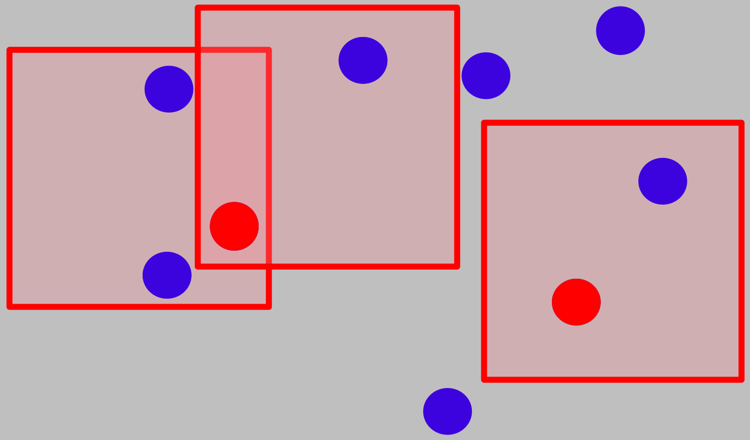
PROBLEM STATEMENT

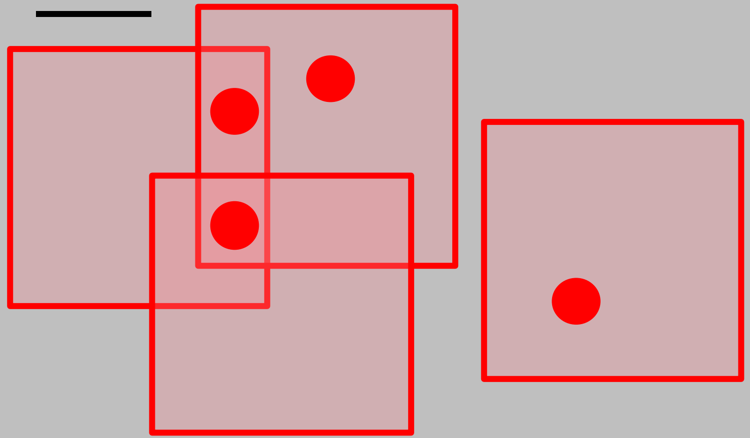
❑ ONLINE HITTING SET PROBLEM

Given: A set $\mathcal{P} \subseteq \mathbb{R}^d$ of points (finite or infinite).
Input: Objects (squares, disks, rectangles) are coming one by one.
Task: Upon the arrival of a new object, maintain a valid hitting set using irrevocable decisions.
Objective: Find a minimum hitting set.

Finite Point Set



Infinite Point Set



COMPETITIVE RATIO

Let $OPT(I)$ be a solution given by an optimal offline algorithm over I.

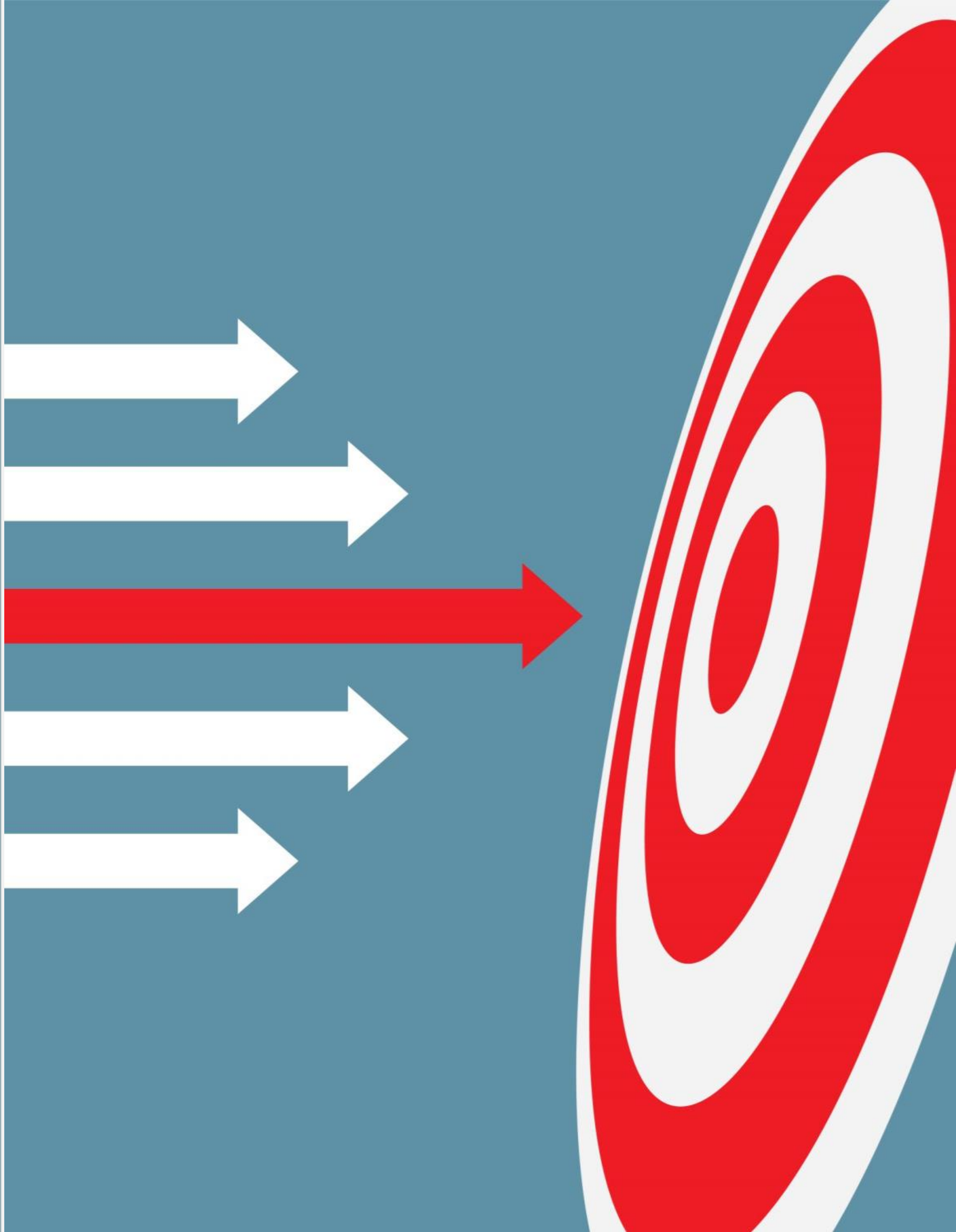
Let $\mathcal{A}(I)$ be the solution given by an online algorithm $\mathcal{A}$ over I.

The *competitive ratio* of the online algorithm $\mathcal{A}$ is defined by,

$$\sup_{I} \frac{\mathcal{A}(I)}{OPT(I)}$$

Idea for Analysis:

We determine the number of points placed by algorithm against each point p in an offline optimum.



RELATED WORK

Infinite Point Set

➢ $\mathcal{P} = \mathbb{R}^d$

- When $\mathcal{S}$ (collection of objcets) consists of intervals, there exists a (deterministic) algorithm that achieves an optimal competitive ratio of $\Theta(n)$.

- De et al. [Algorithmica'24] proposed an algorithm having an competitive ratios of $O(3^d \log M + 2^d)$, when $\mathcal{S}$ consists of hypercubes having side length in $[1, M]$ in $\mathbb{R}^d$.

➢ $\mathcal{P} = \mathbb{Z}^d$

- De and Singh [TCS'24] proposed
  - (deterministic) algorithm that achieves a competitive ratio of 4, when $\mathcal{S}$ consists of unit disks and unit squares.
  - (deterministic) algorithm that achieves a competitive ratio of $O(d^4)$, when $\mathcal{S}$ consists of unit balls.
  - (randomized) algorithm that achieves a competitive ratio of $O(d^4)$, when $\mathcal{S}$ consists of

RELATED WORK

Finite Point Set $|\mathcal{P}| = n$

- When $\mathcal{S}$ consists of intervals, there exists a (deterministic) algorithm that achieves an optimal competitive ratio of $\Theta(n)$.
- Even and Smorodinsky [Discret. Appl. Math.' 14] proposed algorithms having an optimal competitive ratios of $\Theta(\log n)$, when $\mathcal{S}$ consists of half-planes and when $\mathcal{S}$ consists of unit disks in $\mathbb{R}^2$.
- De et al [ESA'25] proposed algorithm having an optimal competitive ratios of $\Theta(\log n)$, when $\mathcal{S}$ consists of disks having radii in $[1, M]$, where M is constant.
- De et al [ArXiv'25] proposed algorithm having an optimal competitive ratios of $\Theta(\log n)$, when $\mathcal{S}$ consists of arbitrary squares.

PROBLEMS TO WORK

Already known but still interesting to solve

1(a). Design $O(n)$-competitive deterministic algorithm when $P = \mathbb{R}$ and $\mathcal{S}$ consists of intervals.
1(b). Design 5-competitive algorithm when $P = \mathbb{R}^2$ and $\mathcal{S}$ consists of disks of radius 1.
1(c). Design 2^d -competitive algorithm when $P = \mathbb{R}^d$ and $\mathcal{S}$ consists of hypercubes of side length 1.

2. Design a 4-competitive algorithm when $P = \mathbb{Z}^2$ and $\mathcal{S}$ consists of unit disks/ squares of side length 2.

3. Design a $O(\log|P|)$ -competitive algorithm when $P \subset \mathbb{R}^2$ is a finite set and $\mathcal{S}$ consists of unit disks/unit squares.

Open problems

4. Design a $O(\log|P|)$ -competitive algorithm when P is finite subset of $\mathbb{R}^2$ and $\mathcal{S}$ consists of arbitrary disks.

5. Design an $o(\log^2|P|)$-competitive algorithm when P is finite subset of $\mathbb{R}^d$ and $\mathcal{S}$ consists of hypercubes in $\mathbb{R}^d$. (open even for $d = 3$)

THANK YOU