Deterministic constructions for pair-isolating families

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Pair-isolating family (PIF)

Input: Universe U of n elements, positive integer k

Output: Family $\mathcal{Q}$ of subsets of U such that for all $x, y, z_1, z_2, \ldots, z_k$, exists $Q \in \mathcal{Q}$ with $x, y \in Q$ and $z_1, z_2, \ldots, z_k \notin Q$

Example: A PIF with $n = 4$, $k = 1$:

$$\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \text{ and } \{2,3,4\}$$

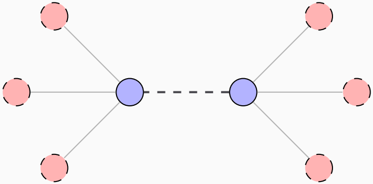
Constructions

- PIF of size $\mathcal{O}(k^3 \log n/k)$ exists by probabilistic argument[1]
- No deterministic polytime construction known
- Pseudorandom (n, k, k^2)-splitters give only a size bound $k^6 \log n$
  - Family of colorings f of U into k^2 colors such that for all subsets S of k elements exists an injective coloring $f|_S$

[1] Christian Konrad, Conor O'Sullivan, Victor Traistaru: Graph Reconstruction via MIS Queries. ITCS 2025.

Motivation

- Consider the GRAPH RECONSTRUCTION problem
- Given vertex set U of a graph G, figure out its hidden structure
- Only indirect access to edges through an oracle
- Oracle outputs a maximal independent set (MIS) of $G[Q]$
- For non-adjacent $x, y \in Q$ whose neighbors are not in Q, both x and y are in all MISs of $G[Q]$



Problems

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Problems:

- Show existence of a PIF of size $\mathcal{O}(k^3 \log n/k)$
- Give an explicit construction of a PIF of size $\mathcal{O}(k^3 \log n/k)$