HIIT Frontier Friday

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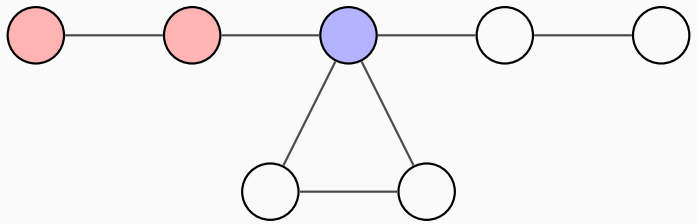
Practicalities

- Workshop-like cohesion-building activity for the local TCS community
- Organized every four weeks (next meetings 14.3., 11.4., and 9.5.)
- Puzzles introducable in 5 minutes, related to research questions
- Possibility of acquiring 1 cr by participating and writing short reports
  - Ask me or Henrik for details

Vertex Integrity

Definition: $\mathrm{vi}(G) := \min_{S \subseteq V} (|S| + |\text{ largest component of } G - S\ |)$

- How easily can we break the graph into small components



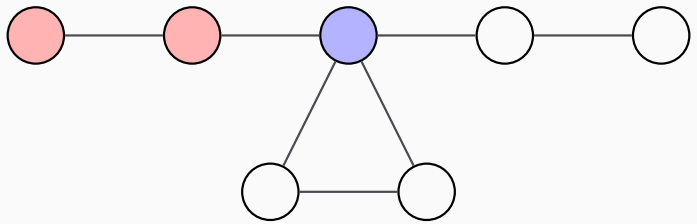
- $\text{treedepth} \leq \text{vi} \leq \text{vertex cover} + 1$
- FO and MSO metatheorems[1]

[1] Michael Lampis, Valia Mitsou. Fine-grained Meta-Theorems for Vertex Integrity. *Log. Methods Comput. Sci.* (2024)

What is known?

- We can test if $\mathrm{vi}(G) \leq k$ in time $k^{O(k)}n^{O(1)}$
- Complexity class known w.r.t. many other graph parameters[2]
- Polytime $O(\log k)$-approximation algorithm[3]
- Related ℓ-Component Order Connectivity cannot be solved in time $2^{o(k \log \ell)}n^{O(1)}$ under ETH:

$$\mathrm{coc}_\ell(G) := \min_{S \subseteq V} |S| \text{ s.t. } |\text{largest component of } G - S| \leq \ell$$



[2] Tesshu Hanaka, Michael Lampis, Manolis Vasilakis, Kanae Yoshiwatari. Parameterized Vertex Integrity Revisited. *MFCS 2024*

[3] Tatsuya Gima, Tesshu Hanaka, Yasuaki Kobayashi, Ryota Murai, Hirotaka Ono, Yota Otachi. Structural parameterizations of vertex integrity. *Theor. Comput. Sci.* (2025)

What is unknown?

For example:

- Algorithm for testing $\mathrm{vi}(G) \leq k$ in time $c^k n^{O(1)}$
- ... or showing its non-existence
- Constant approximation of vi

What next?

$$\mathrm{vi}(G) := \min_{S \subseteq V} \left(|S| + |\,\text{largest component of } G - S\,| \right)$$

- Can we say something new together?
  - Algorithm for testing $\mathrm{vi}(G) \leq k$ in time $c^k n^{O(1)}$
  - ... or showing its non-existence
  - Constant approximation of vi

Thank you for attending!

Feedback: